LIVE

CLASS 12th MATHS

AIM: 500/500 IN BOARDS

BACKLOG MARATHON

SERIES

1. Relations & Functions
2. Inverse Trigonometric Functions
3. Matrices
4. Determinants



BACKLOG MARATHON SERIES

Class 12th | Maths

Relation And Function

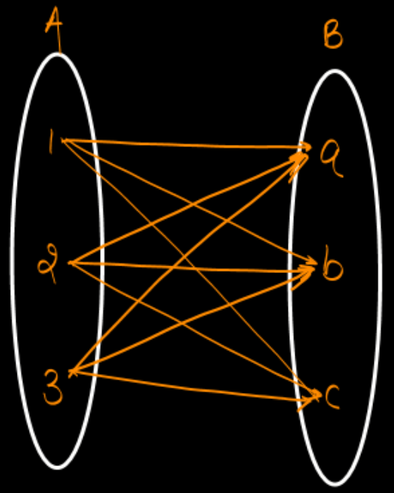
CARTESIAN PRODUCT → All Possible Relations

$A = \{1, 2, 3\}$

$B = \{a, b, c\}$

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c)\}$ ← ROSTER FORM

$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$ → SET BUILDER



(x, y) → ordered Pair

Pre Image of y

Image of x

RELATIONS

→ Spec. Condition

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

Total Possible Relation $= 2^{n(A) \times n(B)}$

$R : A \to B$,

$R = \{(x, y) : y = x + 4 \ \forall \ x \in A \text{ and } y \in B\}$

$R = \{(1, 5), (2, 6)\}$

$y = x + 4$

$y = 1 + 4 = 5$

$y = 2 + 4 = 6$

Domain(R) $= \{1, 2\}$

Range (R) $= \{5, 6\}$

Codomain(R) $= \{4, 5, 6\}$

$A = \{1, 2, 4\}$

$B = \{2, 3, 4, 5, 6\}$

$R = \{(1, 2), (2, 3), (4, 5)\}$

Domain $= \{1, 2, 4\}$

Range $= \{2, 3, 5\}$

Codomain $= \{2, 3, 4, 5, 6\}$

BACKLOG MARATHON SERIES

Next Toppers



Types of Relations

$R: \{(1,2),(2,3)\}$

$A = \{1,2,3\}$

$A: \{1,2,3\}$

$B: \{4,5,6\}$

Empty Relation

$R = \{\} = \phi$

$R: \{(x,y) : x > y \ \forall x \in A \ \& \ y \in B\}$

$R = \phi$

Universal Relations

$R = A \times B$

Reflexive Relation

Selfish $(x,x) \in R$

Set A के सभी Elements Ka Relation Apne Sath

$R_1 = \{(1,1),(2,2),(3,3)\}$

#GKB

$R = \{(1,2),(2,3)\}$

$(1,2) \in R$, $(2,3) \in R$

1 R 2 , 2 R 3

Symmetric Relation

Tou dou

Agar (x,y) है तो (y,x) bhi hona chahiye

$R_2 = \{(1,2),(2,1)\}$

#GKB

Agar koi Relation Reflexive, Symmetric Transitive ho to use Equivalence Rel^N

Transitive Relation

Dhoka pe Dhoka

Agar (x,y) & (y,z) है तो (x,z) bhi hona chahiye

Agar $(x,y) \in R$ & $(y,z) \notin R$ तो $(x,z) \notin R$

$R_1: \{(1,2),(2,3),(1,3)\}$ ✓ (T T T)

$R_2 = \{(1,2),(2,1),(1,1)\}$ ✗ (T ⊗)

$R_3 = \{(1,2)\}$ ✓

$R_4 = \{(1,2),(2,3),(1,3),(3,1),(1,1)\}$ ✗ (⊕ ×)

Reflexive Relⁿ

$A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$

$R_2 = \{(1,1), (2,2)\}$ ✗

$(3,3) \notin R$

$R_3 = \{(1,1), (2,2), (3,3), (1,2)\}$ ✓

Symmetric

↳ $(x, y) \in R \Rightarrow (y, x)$

$R = \{(1,2), (2,1)\}$ —

$R = \{(2,3), (3,2), (1,2)\}$

$(2,1) \notin R$

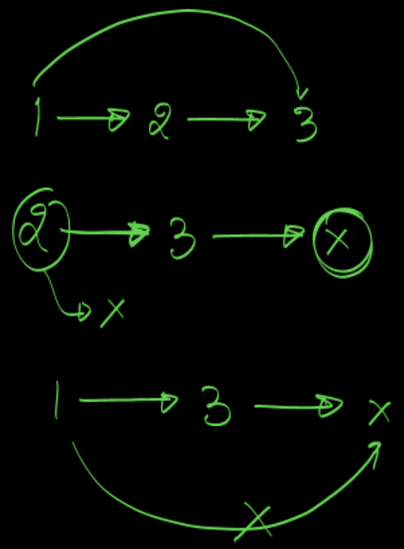
Equivalence Relⁿ

TRANSITIVE

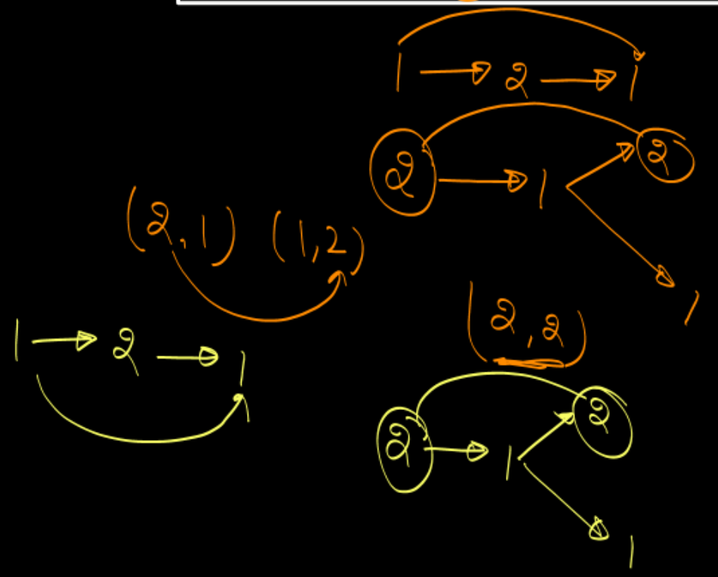
$(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$

$(x,y) \in R \;\&\; (y,z) \notin R \Rightarrow (x,z) \notin R$

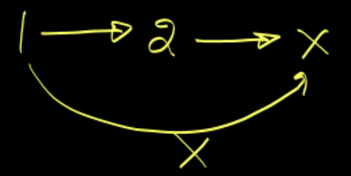
$R_1 = \{(1,2), (2,3), (1,3)\}$ T ✓



$R_2 = \{(1,2), (2,1), (1,1)\}$ T X

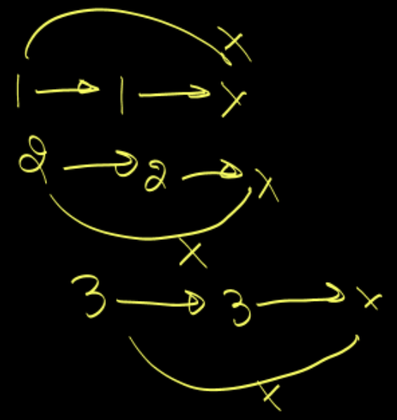


$R_3 = \{(1,2)\}$ ✓



$R_4 = \{(1,1), (2,2), (3,3)\}$







FUNCTIONS

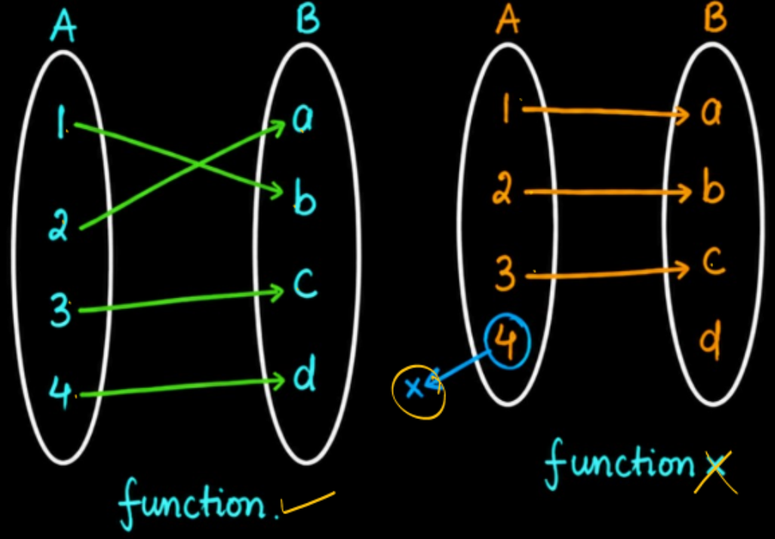
$f: A \to B$

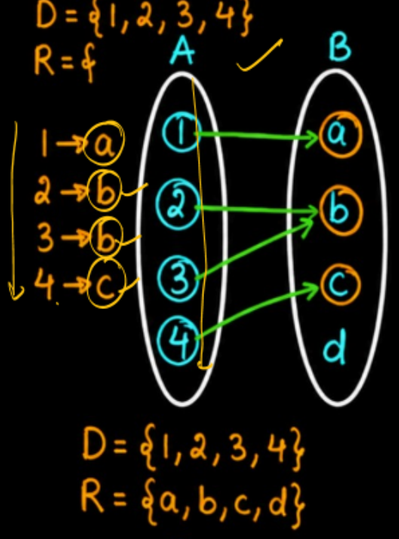
$f: A \xrightarrow{\text{Domain}} B$ (Codomain)

$A = \{1, 2, 3, 4\}$

$B = \{a, b, c, d\}$

Aisa Relation Jab Set A के सबके Elements ka Sirf Ek Image Set B में ho

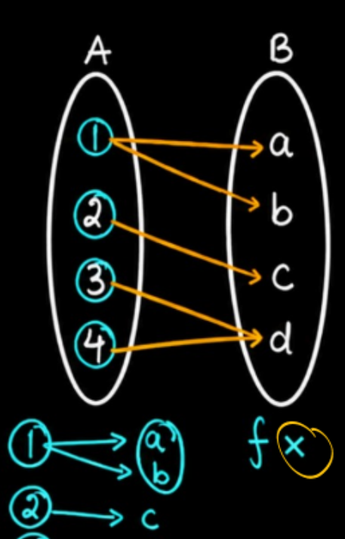




$D = \{1, 2, 3, 4\}$

$R = \{a, b, c, d\}$

$C.D = \{a, b, c, d\}$



1 → a, b

2 → c

3 → d

4 → d



f ✗

BACKLOG MARATHON SERIES

Next Toppers



1 → a
2 → b
3 → c

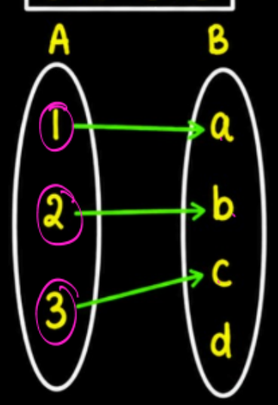
1 → a
2 → a
3 → b

TYPES OF FUNCTION

INT. + SUR. = BIJECTIVE

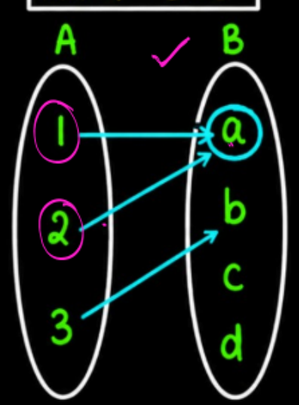
INJECTIVE

ONE-ONE



* Ek Element ka Image and wo image kisi or Element ka na ho

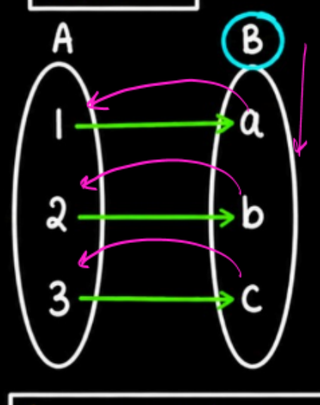
MANY-ONE



Ek Element ka Jo Image hai wo kisi or Element ka bhi Image ho

SURJECTIVE

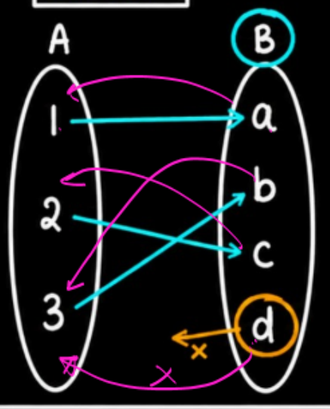
ONTO



Agar Set B ke sare Elements ki pre-image Hai

→ RANGE = CODOMAIN

INTO



Agar Set B ke kisi Element ki koi pre-image na ho

→ RANGE ≠ CODOMAIN



R ✓

Show that the relation R on the set A = {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor trans.

$(1,2) \in R$

$(2,1) \notin R$

↓

Not Symm.

1 → 2 → 3

$(1,3) \notin R$

↓

not Trans.



$x, y \Rightarrow |x-y|$

$a, b \Rightarrow |a-b|$ $\quad (y,x) = |y-x|$

$|x| = a \Rightarrow x = \pm a$

Show that the relation Ron the set $A = \{x \in \mathbb{Z} \mid 0 \le x \le 12\}$ given by $R = \{(a, b) : |a-b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1 i.e. equivalence class [1].

$A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$R = \{(a,b) : |a-b| = 4m\}$

REFLEXIVE

$x \in A$

(x, x)

$R: |x-x| = 0$

$(x, x) \in R$

Hence R is Ref.

SYMMETRIC

Let $(x, y) \in R$

$R: |x-y| = 4m$

$R: |-(y-x)| = 4m$

$R: |y-x| = 4m$

$\Rightarrow (y, x) \in R$

Hence R is Symm.

TRANSITIVE

Let $(x,y) \in R$ and $(y,z) \in R$

$\Rightarrow |x-y| = 4m$ & $|y-z| = 4n$

$x-y = \pm 4m$ & $y-z = \pm 4n$

$x-y+y-z = \pm 4m \pm 4n$

$x-z = \pm 4(m+n) = \pm 4p$

$|x-z| = 4p$

$(x, z) \in R$

Hence R is Transitive.

∴ R is an Equivalence Relation.

Equivalence Class

$[1] = \{1, 5, 9\}$

$[2] = \{2, 6, 10\}$

$[0] = \{0, 4, 8, 12\}$



Prove that $f: R \to R$, given by $f(x) = 2x$ is one-one and onto.

ONE-ONE

Let x_1 & $x_2 \in R$ (Domain)

Such that $f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$x_1 = x_2$

One-one ✓

ONTO

$y \in R$ (CODOMAIN)

$y = f(x) = 2x$

$\frac{y}{2} = x$

$f(x) = 2x$

$f\left(\frac{y}{2}\right) = 2\left(\frac{y}{2}\right) = y$

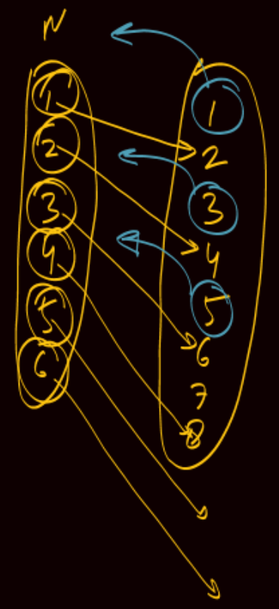
Hence Every Element in Codomain have some pre-Image In Domain

$\therefore f$ is onto

$f: R \to R$ $\quad f(x) = 2x$

$f: N \to N$ $\quad f(x) = 2x$

one-one ✓ $\quad$ onto ✗



I don't have any pre image.

one-one ✓ Variable.

one-one ✗ Example.

Sat. → 6:30pm



Let $A = R - \{2\}$ and $B = R - \{1\}$ If $f : A \to B$ is a mapping defined by $f(x) = \dfrac{x-1}{x-2}$, show that f is bijective .

Let $(x_1, x_2) \in A$

Such that $f(x_1) = f(x_2)$

$$\frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$$

$$(x_1 - 1)(x_2 - 2) = (x_2 - 1)(x_1 - 2)$$

$$x_1x_2 - 2x_1 - x_2 + 2 = x_2x_1 - 2x_2 - x_1 + 2$$

$$-2x_1 + x_1 = -2x_2 + x_2$$

$$-x_1 = -x_2$$

$$x_1 = x_2$$ → One-one

Let $y \in B$

$$y = \frac{x-1}{x-2}$$

$$y(x-2) = x - 1$$

$$yx - 2y = x - 1$$

$$yx - x = -1 + 2y$$

$$x(y-1) = 2y - 1$$

$$x = \frac{2y-1}{y-1}$$ → Range

$y - 1 \neq 0$

$y \neq 1$

$y \in R - \{1\}$

Range : Codomain

onto ✓



Show that a function f: R→R defined by $f(x) = \dfrac{2x}{1+x^2}$ is neither one-one nor onto.

Further, find set A so that the given function f: R→ A becomes an onto function.

Let $x_1, x_2 \in R$ (Domain)

Such that $f(x_1) = f(x_2)$

$$\frac{2x_1}{1+x_1^2} = \frac{2x_2}{1+x_2^2}$$

$$x_1(1+x_2^2) = x_2(1+x_1^2)$$

$$x_1 + x_1x_2^2 = x_2 + x_2x_1^2$$

$$(x_1 - x_2) + x_1x_2^2 - x_2x_1^2 = 0$$

$$(x_1 - x_2) - x_1x_2(-x_2 + x_1) = 0$$

$$(x_1 - x_2) - x_1x_2(x_1 - x_2) = 0$$

$$(x_1 - x_2)(1 - x_1x_2) = 0$$

$x_1 - x_2 = 0$

$x_1 = x_2$

$1 - x_1x_2 = 0$

$x_1x_2 = 1$

$x_1 = \frac{1}{x_2}$

$x_1 \neq x_2$

Not one-one

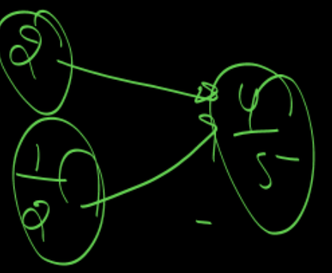
$x = 2$

$$f(x) = \frac{2(2)}{1+2^2} = \frac{4}{1+4} = \frac{4}{5}$$

$x = \frac{1}{2}$

$$f(x) = \frac{2 \times \frac{1}{2}}{1+\left(\frac{1}{2}\right)^2} = \frac{1}{1+\frac{1}{4}} = \frac{1}{\frac{4+1}{4}}$$

$$= \frac{4}{5}$$





$f: R \to A$ (A: Codomain)

Show that a function f: R→R defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto.

Further, find set A so that the given function f: R→ A becomes an onto function.

$A = [-1, 1] - \{0\}$.

Let $y \in R$ (Codomain)

$y = \frac{2x}{1+x^2}$

$y(1+x^2) = 2x$

$y + yx^2 = 2x$

$\boxed{yx^2 - 2x + y = 0}$

$ax^2 + bx + c = 0$

$a = y,\ b = -2,\ c = y$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{4 - 4y \times y}}{2y}$

$x = \frac{2 \pm \sqrt{4(1-y^2)}}{2y}$

$x = \frac{2 \pm 2\sqrt{1-y^2}}{2y}$

$x = \frac{2(1 \pm \sqrt{1-y^2})}{2y}$

$\boxed{x = \frac{1 \pm \sqrt{1-y^2}}{y}}$ → Range

$y \neq 0 \Rightarrow y \in R - \{0\}$

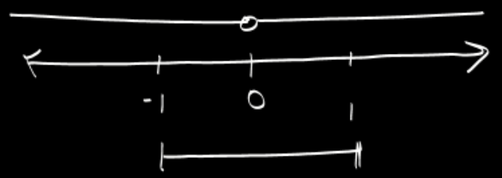
$1 - y^2 \geq 0$

$(1-y)(1+y) \geq 0$

$\boxed{y \in [-1, 1]}$

$y \in [-1, 0) \cup (0, 1]$

$y \in [-1, 1] - \{0\}$ → Range



Range ≠ Codomain

Not Onto



Consider the function f: $[0, \pi/2] \to R$ given by $f(x) = \sin x$ and g: $[0, \pi/2] \to R$ given by $g(x) = \cos x$. Show that fand g are one-one, but f + g is not one-one.

BACKLOG MARATHON SERIES

Class 12th | Maths

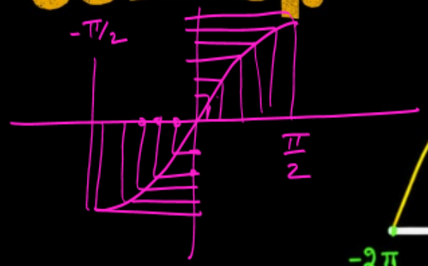
Inverse Trigonometric Functions

Chapter 2

BACKLOG MARATHON SERIES

Next Toppers

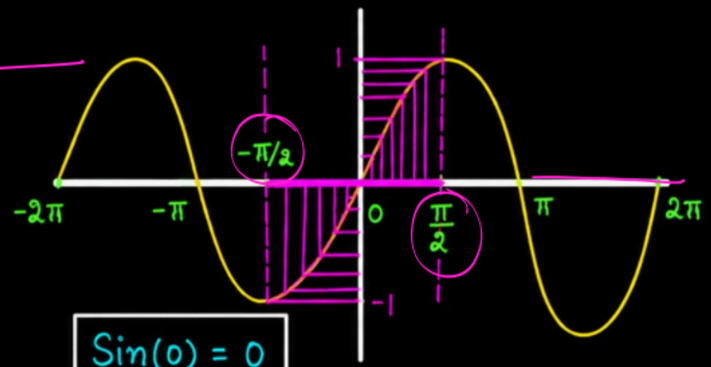




$f(x) = \sin x$

Domain = R

Range : $[-1, 1]$



$\sin(0) = 0$

$\sin(\pi) = 0$

$\sin(2\pi) = 0$ → $\sin x$ is not one-one

$\sin x$ Ka Inverse Nahi Nikal Sakte ✗

BASICS

$f(x)$ → Agar Koi $f(x)$ one-one and onto hoga to uska Inverse Exist Karega

Bijective

Agar $\sin x$ Ki Domain Ko $[-\frac{\pi}{2}, \frac{\pi}{2}]$ tak Restrict Kar du

Ab $\sin x$ one-one hoga ⇒ Inverse Nikal Sakte hai

$f(x) = \sin x$

Domain = $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Range : $[-1, 1]$

Inverse

$\sin^{-1} x$

Domain = $[-1, 1]$

Range : $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Principle Value

GKB

$\sin^{-1} x \neq (\sin x)^{-1}$

$(\sin x)^{-1} = \frac{1}{\sin x}$

1 → a

2 → b

3 → c

4 → d

fx



PRINCIPAL VALUE

ITF	DOMAIN	RANGE
$\sin^{-1}x$	$[-1,1]$	$[-\frac{\pi}{2},\frac{\pi}{2}]$
$\cos^{-1}x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1}x$	R	$(-\frac{\pi}{2},\frac{\pi}{2})$
$\cot^{-1}x$	R	$(0,\pi)$
$\operatorname{cosec}^{-1}x$	$R-(-1,1)$	$[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$
$\sec^{-1}x$	$R-(-1,1)$	$[0,\pi]-\{\frac{\pi}{2}\}$

Range

$f(x) = \sin^{-1}(x) = (y)$

Domain

$\sin^{-1}(\sin x) = x$

$\cos^{-1}(\cos x) = x$

$\tan^{-1}(\tan x) = x$

$\cot^{-1}(\cot x) = x$

$\sec^{-1}(\sec x) = x$

$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$

$\sin(\sin^{-1}x) = x$

$\cos(\cos^{-1}x) = x$

$\tan(\tan^{-1}x) = x$

$\cot(\cot^{-1}x) = x$

$\sec(\sec^{-1}x) = x$

$\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$



ITF (+ve) → No Tension

$\sin^{-1}(-x) = -\sin^{-1}x$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

BACKLOG MARATHON SERIES

Next Toppers



PROPERTIES

$\sin^{-1}(\sin x) = x \quad , x \in [-\pi/2, \pi/2]$

$\cos^{-1}(\cos x) = x \quad , x \in [0, \pi]$

$\tan^{-1}(\tan x) = x \quad , x \in (-\pi/2, \pi/2)$

$\cot^{-1}(\cot x) = x \quad , x \in (0, \pi)$

$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad , x \in [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

$\sec^{-1}(\sec x) = x \quad , x \in [0, \pi] - \{\pi/2\}$

$\sin(\sin^{-1} x) = x \quad , x \in [-1, 1]$

$\cos(\cos^{-1} x) = x \quad , x \in [-1, 1]$

$\tan(\tan^{-1} x) = x \quad , x \in R$

$\cot(\cot^{-1} x) = x \quad , x \in R$

$\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x \quad , x \in R - (-1, 1)$

$\sec(\sec^{-1} x) = x \quad , x \in R - (-1, 1)$

$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$

$2\tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$

$\sin(\pi - \theta) = \sin\theta$

$\operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\sec(\pi - \theta) = -\sec\theta$

$\tan(\pi - \theta) = -\tan\theta$

$\cot(\pi - \theta) = -\cot\theta$

$\sin^2 x + \cos^2 x = \frac{\pi}{2}$

$\tan^2 x + \cot^2 x = \frac{\pi}{2}$

$\operatorname{cosec}^2 x + \sec^2 x = \frac{\pi}{2}$

$\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1} x \quad , x \geq 1 \text{ or } x \leq -1$

$\cos^{-1}\frac{1}{x} = \sec^{-1} x \quad , x \geq 1 \text{ or } x \leq -1$

$\tan^{-1}\frac{1}{x} = \cot^{-1} x \quad , x > 0$

BACKLOG MARATHON SERIES

Next Toppers

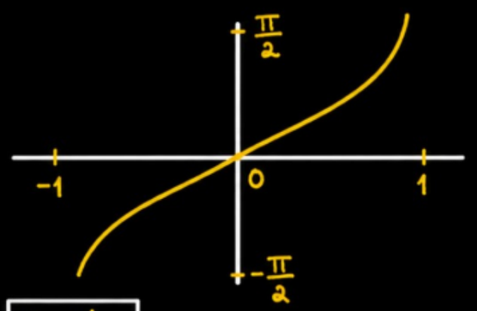


GRAPHS

Domain → x-Axis
Range → y-Axis

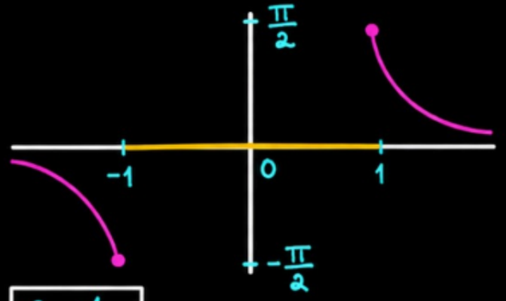
$\sin^{-1}x$

$D = [-1,1]$
$R = [-\frac{\pi}{2}, \frac{\pi}{2}]$



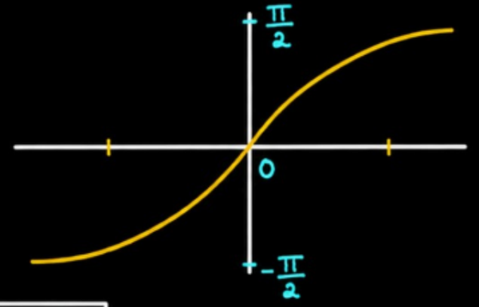
$\text{Cosec}^{-1}x$

$D = R - \{-1,1\}$
$R = [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$



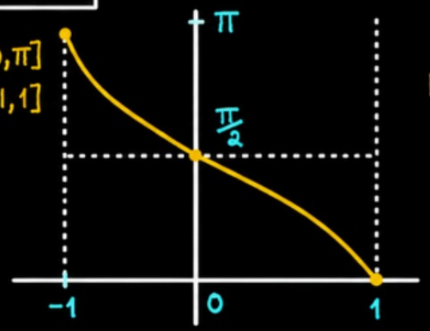
$\tan^{-1}x$

$D = R$
$R = (-\frac{\pi}{2}, \frac{\pi}{2})$



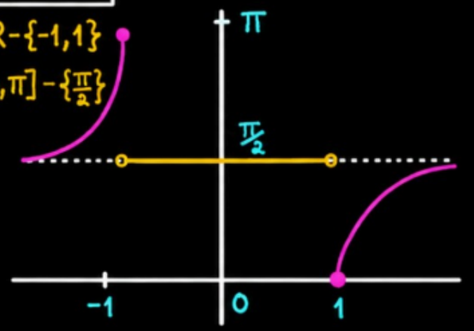
$\cos^{-1}x$

$D = [0,\pi]$
$R = [-1,1]$



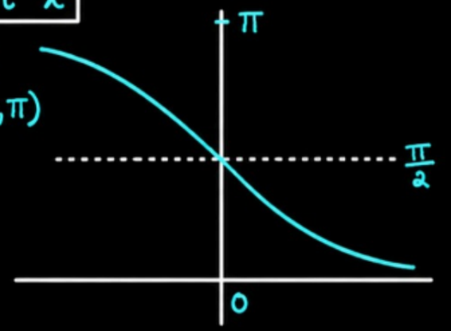
$\text{Sec}^{-1}x$

$D = R - \{-1,1\}$
$R = [0,\pi] - \{\frac{\pi}{2}\}$



$\cot^{-1}x$

$D = R$
$R = (0,\pi)$





$\sin^{-1}(-x) = -\sin^{-1}(x)$

Range (y)

Find the principal values of :-

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$

$y = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$y = \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$y = \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$-\sin^{-1}\left(\sin\frac{\pi}{6}\right) = -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Find the value of :-

$$\sin^{-1}\left[\cos\left\{\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}\right]$$

$\cos(-\theta) = \cos\theta$

$-\frac{\pi}{3}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow -\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$\Rightarrow \left(-\frac{\pi}{3}\right)$

$\sin^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

$\sin^{-1}\left(\cos\frac{\pi}{3}\right)$

$\sin^{-1}\left(\frac{1}{2}\right)$

$\sin^{-1}\left(\sin\frac{\pi}{6}\right)$

$\left(\frac{\pi}{6}\right)$



Find the domain of $\cos^{-1}(2x-1)$.

$\cos^{-1}(\theta)$

$-1 \le \theta \le 1$

$\cos^{-1}(2x-1)$

$x \in [0,1] \rightarrow$ Domain

$-1 \le 2x-1 \le 1$

$-1+1 \le 2x-1+1 \le 1+1$

$\frac{0}{2} \le \frac{2x}{2} \le \frac{2}{2}$

$0 \le x \le 1$



For the principal values, evaluate each of the following:-

(i) tan-1 $\sqrt{3} - sec^{-1}(-2) + cosec^{-1}\frac{2}{\sqrt{3}}$

(ii) 2sec-1 (2)-2cosec-1 (-2)

$\cos(\pi+\theta) = -\cos\theta$ $\quad \cos^{-1}(-x) = \pi - \cos^{-1}x$



Evaluate each of the following: $\quad \tan^{-1}(-x) = -\tan^{-1}x$

(i) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(ii) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) \Rightarrow -\frac{\pi}{4}$

(iii) $\sin^{-1}(\sin(-600°))$

(iv) $\cos^{-1}(\cos(-600°))$

$\cos\frac{7\pi}{6} = \cos\left(\pi+\frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6}$

$\cos^{-1}\left(-\cos\frac{\pi}{6}\right) = \pi - \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6} \in [0,\pi]$

$\tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4}$

$\tan^{-1}\left(-\tan\frac{\pi}{4}\right) = -\tan^{-1}\left(\tan\frac{\pi}{4}\right) = -\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$\sin(-\theta) = -\sin\theta$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$



Evaluate each of the following:

(i) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(ii) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

(iii) $\sin^{-1}(\sin(-600°))$

(iv) $\cos^{-1}(\cos(-600°))$

600°

$180° = \pi$

$1° = \frac{\pi}{180}$

$600 = \frac{10\cancel{0}}{\cancel{600}} \times \frac{\pi}{\cancel{20}_{3}} = \frac{10\pi}{3}$

$\sin^{-1}\left[\sin(-600°)\right]$

$\sin^{-1}\left[\sin\left(-\frac{10\pi}{3}\right)\right]$

$\sin^{-1}\left[-\sin\frac{10\pi}{3}\right]$

$\sin^{-1}\left[-\sin\left(3\pi+\frac{\pi}{3}\right)\right]$

$\sin^{-1}\left[\sin\left(\pi+\frac{\pi}{3}\right)\right]$

$\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$\frac{\pi}{3}$

$= \cos^{-1}\left(-\cos\pi/3\right)$

$\begin{vmatrix} \cos(600) \\ \cos\left(3\pi+\frac{\pi}{3}\right) \\ -\cos(\pi/3) \end{vmatrix} =$



Express $\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$ in the simplest form.

ITF (TF)

ITF (Variable)

$\cos 2\theta$: $\cos^2\theta - \sin^2\theta$

$\sin 2\theta$: $2\sin\theta\cos\theta$

$$\frac{\cos x}{1-\sin x}$$

$$\frac{\cos^2 x/2 - \sin^2 x/2}{\cos^2 x/2 + \sin^2 x/2 - 2\sin x/2 \cos x/2}$$

$$\frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 - \sin x/2)^2}$$

$$\frac{(\cos x/2 - \sin x/2)(\cos x/2 + \sin x/2)}{(\cos x/2 - \sin x/2)^2}$$

$$\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}$$

$$\frac{1 + \tan x/2}{1 - \tan x/2 \times 1}$$

$$\frac{\tan \frac{\pi}{4} + \tan x/2}{1 - \tan \frac{\pi}{4} \tan x/2}$$

$$\Rightarrow \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$



ITF (TF)

ITF (Variable)

Express $\tan^{-1}\dfrac{\cos x}{1-\sin x}, -\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$ in the simplest form.

$\cos 2\theta : \cos^2\theta - \sin^2\theta$

$\sin 2\theta : 2\sin\theta\cos\theta$

$\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$

$\frac{\pi}{4}+\frac{x}{2}$



$\frac{1}{2}\tan^{-1}x$

Write the following functions in the simplest form:

$$\frac{\tan^{-1}(\sqrt{1+x^2}-1)}{x}, \quad x \neq 0$$

$1+x^2$

$1+\tan^2\theta = \sec^2\theta$

$1+\cot^2\theta = \operatorname{cosec}^2\theta$

$x = \tan\theta \Rightarrow \tan^{-1}x = \theta$

$$\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$$

$$\tan^{-1}\left[\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right]$$

$$\tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$$

$$\tan^{-1}\left[\tan\theta/2\right] = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

$$\frac{\sec\theta-1}{\tan\theta}$$

$$\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\frac{1-\cos\theta}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}} = \frac{1-\cos\theta}{\sin\theta}$$

$$\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}$$

$$\frac{\sin\theta/2}{\cos\theta/2} \quad \tan\theta/2$$



Write the following functions in the simplest form:

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0; \frac{-a}{3} \leq x \leq \frac{a}{3}$$



$\alpha = \tan^{-1}x \qquad \beta = \tan^{-1}y$

Find the values of each of the following:-

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|0 \text{ and } xy < 1$$

$x = \tan\alpha \qquad y = \tan\beta$

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2\tan\alpha}{1+\tan^2\alpha} + \cos^{-1}\frac{1-\tan^2\beta}{1+\tan^2\beta}\right]$$

$$\tan\frac{1}{2}\left[\sin^{-1}(\sin 2\alpha) + \cos^{-1}(\cos 2\beta)\right]$$

$$\tan\frac{1}{2}\left[2\alpha + 2\beta\right]$$

$$\tan\frac{1}{2}\left[2(\alpha+\beta)\right]$$

$$\tan(\alpha+\beta)$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\frac{\tan(\tan^{-1}x) + \tan(\tan^{-1}y)}{1 - \tan(\tan^{-1}x)\tan(\tan^{-1}y)}$$

$$\frac{x+y}{1-xy}$$



Prove the following:

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x \in [0, 1]$$

$x = \tan^2\theta \Rightarrow \sqrt{x} = \tan\theta$

$\tan^{-1}\sqrt{x} = \theta$

$\frac{1}{2}\cos^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$

$\frac{1}{2}\cos^{-1}[\cos 2\theta]$

$\frac{1}{2} \times 2\theta = \theta = \tan^{-1}\sqrt{x}$



Prove the following:

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$$

$$\frac{2+2\sqrt{\cos^2 x}}{2\sin x}$$

$$\frac{2(1+\cos x)}{2\sin x}$$

$$\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\times\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$$

$$\frac{\left[\sqrt{1+\sin x}+\sqrt{1-\sin x}\right]^2}{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2}=\frac{\left(\sqrt{1+\sin x}\right)^2+\left(\sqrt{1-\sin x}\right)^2+2\left(\sqrt{1+\sin x}\right)\left(\sqrt{1-\sin x}\right)}{(1+\sin x)-(1-\sin x)}$$

$$\Rightarrow\frac{1+\sin x+1-\sin x+2\sqrt{1^2-\sin^2 x}}{1+\sin x-1+\sin x}$$



Prove the following:

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$$

$$\cot^{-1}\left[\frac{1+\cos x}{\sin x}\right]$$

$$\cot^{-1}\left[\frac{2\cos^2 x/2}{2\sin x/2\cos x/2}\right]$$

$$\cot^{-1}\left[\frac{\cos x/2}{\sin x/2}\right]$$

$$\cot^{-1}\left[\cot x/2\right]=x/2$$

BACKLOG MARATHON SERIES

Class 12th | Maths

Matrices

Chapter 3

BACKLOG MARATHON SERIES

Next Toppers



MATRICES

Is an Arrangement of Elements in A Rectangular Array

Rows, Columns

K·B →	2C	3B	6C
S·N →	4C	6B	3C
P·K →	5C	3B	5C
DSR →	6C	7B	10C

	C	B	Cy	
K·B	2	3	6	→ R_1
S.N	4	6	3	→ R_2
P.K	5	3	5	→ R_3
DSR	6	7	10	→ R_4
	↓ C_1	↓ C_2	↓ C_3	4 × 3
				R C

a_{11}

1st R 1st C | 1&2C | 1&3C | 1&4C

K·B S·N P·K DSR

C → 2 4 5 6 → R_1

B → 3 6 3 7 → R_2

Cy → 6 3 5 10 → R_3

C_1 C_2 C_3 C_4

3 × 4

Rows = 3

Column = 4

a_{32} (R C)

a_{33}

3R3C

$a_{21} : 3$

$a_{34} = 10$

Order = 3 × 4 (R × C)

No. of Element = 12



Representation of a Matrix

$m \rightarrow$ Rows
$n \rightarrow$ Columm

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$A = [a_{ij}]_{m \times n}$

Elements / Entries of Matrix

No. of Row

No. of Column

$1 \le i \le m$
$1 \le j \le n$

$(m, n \in N)$

Order of a Matrix $\rightarrow m \times n$

No. of Elements $\rightarrow mn$

Adress of An Element

$a_{21} = 3$, $a_{22} = 3$, $a_{33} = 5$



Equality of Matrices

- 2 Matrix Equal Kab Hoti Hai
  - Jab Order Same Ho
  - Corresponding Elements Equal Ho

$$A = \begin{bmatrix} x & z \\ y & w \end{bmatrix}_{2\times 2}$$

$$B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}_{2\times 2}$$

$A = B$

$x = 1 \quad z = 3$

$y = 2 \quad w = 4$

ROW MATRIX → $[1\ 2\ 3]_{1\times 3}$

COLUMN MATRIX $\begin{bmatrix}1\\2\\3\end{bmatrix}_{3\times 1}$

RECTANGULAR → $m \neq n$

SQUARE MATRIX $m = n$

DIAGONAL MATRIX

$$\begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 6\end{bmatrix}$$

Diagonal Element

$i = j$

SCALAR

$$\begin{bmatrix}4 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix}$$

IDENTITY

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

ZERO

$$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$6-(-3)$

$$A: \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}_{2\times 3}$$

$$B: \begin{bmatrix} 1 & 3 & -3 \\ 0 & -2 & 4 \end{bmatrix}_{2\times 3}$$

$$A+B = \begin{bmatrix} 3 & 7 & 3 \\ 3 & 3 & 11 \end{bmatrix}$$

$$A-B: \begin{bmatrix} 1 & 1 & 9 \\ 3 & 7 & 3 \end{bmatrix}_{2\times 3}$$

$$A: \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$2A = \begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix}$$

$A_{m\times n}$ $B_{p\times q}$ $\Rightarrow (AB)_{m\times p}$

$n = p$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 6 \end{bmatrix}_{2\times 3}$$

$$AB: \begin{bmatrix} 0-6 & 1+9 & 2+18 \\ 0-8 & 2+12 & 4+24 \end{bmatrix}$$

$$AB: \begin{bmatrix} -6 & 10 & 20 \\ -8 & 14 & 28 \end{bmatrix}$$



Transpose of Matrix

Row ⟷ Column

$A' = A^T$

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 2 & -1 & -6 \\ 3 & 4 & -2 \end{bmatrix} \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{matrix}$$

(C_1, C_2, C_3 mark the columns)

$$A' = B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 5 & -6 & -2 \end{bmatrix}$$

A Matrix Ka Transpose

PROPERTIES

$(A')' = A$

$(KA)' = K A'$

$\star$ $(AB)' = B' A'$

$(A \pm B)' = A' \pm B'$

$(ABC)' = C' B' A'$

$A = A'$ → Symmetric

$A = -A'$ → Sk. Symm

(Diagonal Elements = 0)

$A + A' = P$ → Symm.

$A - A' = Q$ → Sk. Symm.



2×2

Construct a ~~3 x 4~~ matrix A = [a] whose elements a_{ij} are given by:

(i) $a_{ij} = i + j$ (ii) $a_{ij} = i - j$ (iii) $a_{ij} = 2i$

(iv) $a_{ij} = j$ (v) $a_{ij} = \frac{1}{2}|-3i+j|$

$A = [a_{ij}]_{2\times 2}$ $\quad a_{ij} = \frac{1}{2}|-3i+j|$ $\longrightarrow$ $A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$

$a_{11} = \frac{1}{2}|-3(1)+1| = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{1}{2}\times 2 = 1$

$a_{12} = \frac{1}{2}|-3(1)+2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$

$a_{21} = \frac{1}{2}|-3(2)+1| = \frac{1}{2}|-6+1| = \frac{5}{2}$

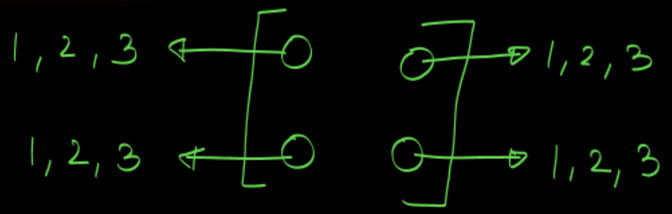
$a_{22} = \frac{1}{2}|-3(2)+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$

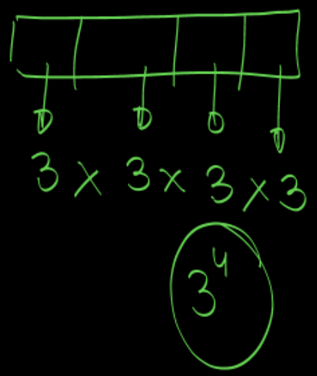


Write the number of all possible matrices of order 2 x 2 with each entry 1, 2 or 3.

81

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$







$x = 3, y = 1$

Find the values of x and y if

$$\begin{bmatrix} X+10 & Y^2+2Y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

$x + 10 = 3x + 4$

$6 = 3x - x$

$6 = 2x$

$3 = x$

$y^2 + 2y = 3$

$y^2 + 2y - 3 = 0$

$y^2 + 3y - y - 3 = 0$

$y(y+3) - 1(y+3) = 0$

$(y+3)(y-1) = 0$

$y = -3$ $\quad y = 1$

$y^2 - 5y = -4$

$y^2 - 5y + 4 = 0$

$y^2 - 4y - y + 4 = 0$

$y(y-4) - 1(y-4) = 0$

$(y-4)(y-1) = 0$

$y = 4$ $\quad y = 1$



Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

$$2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$



Find x, y, z and t, if

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$2x + 3 = 9$

$2x = 6$

$x = 3$

$2z - 3 = 15$

$2z = 18$

$z = 9$

$2y = 12$

$y = 6$

$2t + 6 = 18$

$2t = 12$

$t = 6$



$I - E = S$

$A = 3 \times 30K = 90K$

$B = 4 \times 30K = 120K$

The monthly incomes of Aryan and Babban are in the ratio 3: 4 and their monthly expenditures are in the ratio 5: 7. If each saves ₹15000 per month, find their monthly incomes using matrix method. This problem reflects which value?

Let Monthly Income of Aryan $3x$

" " " Babban $4x$

" Exp. " Aryan $5y$

" " " Babban $7y$

" Saving " " 15000

" " " Aryan 15000

$$I = \begin{bmatrix} 3x \\ 4x \end{bmatrix} \quad E = \begin{bmatrix} 5y \\ 7y \end{bmatrix} \quad S = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$$\begin{bmatrix} 3x - 5y \\ 4x - 7y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$3x - 5y = 15000$

$4x - 7y = 15000$

$x = 30K$

$y = 15K$



If find $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ $A^2-5A+4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$.

$$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$5A = \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$$

$$4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^2 - 5A + 4I$$

$$\begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$A^2 - 5A + 4I + X = O$$

$$\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix} + X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$$



If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A^T - B^T$.

$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$A' - B' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$



If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix, find x, y, z and t.

$$A' = \begin{bmatrix} 5 & y & 4 \\ 2 & z & t \\ x & -3 & -7 \end{bmatrix} = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$$

$y = 2$

$x = 4$

$z = z$ → Any no.

$t = -3$



If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to ?

$$(I+A)^3 - 7A$$

$$I^3 + A^3 + 3I^2A + 3IA^2 - 7A$$

$$I + A^2A + 3IA + 3IA^2 - 7A$$

$$I + A \cdot A + 3A + 3IA - 7A$$

$$I + A^2 + 3A + 3A - 7A$$

$$I + A + 3A + 3A - 7A$$

$$I$$

$A = \frac{1}{2}(P + Q)$ (P → Symm., Q → Sk. Symm.)



Express the matrix $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix and verify your result.

$A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$P = A + A' = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

$P' = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = P \rightarrow$ Symm.

$Q = A - A' = \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = -Q \rightarrow$ Sk. Symm.

$\frac{1}{2}(P + Q) = \frac{1}{2}\begin{bmatrix} 6 & -4 & -8 \\ 6 & -4 & -10 \\ -2 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$

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Class 12th | Maths

Determinants

Chapter 4

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DETERMINANTS

|A|, Δ

A No. Associated With a Square Matrix

Row = Column

$A_{1\times1}$ $A_{2\times2}$ $A_{3\times3}$

$A_{1\times1} = [4] \Rightarrow |A| = |4| = 4$

$A = [-3] = |A| = |-3| = -3$

$A_{2\times2} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} \Rightarrow 4 - 6 = -2$

$A = \begin{bmatrix} 2 & 4 \\ -1 & -5 \end{bmatrix} \Rightarrow \begin{vmatrix} 2 & 4 \\ -1 & -5 \end{vmatrix} \Rightarrow -10 - (-4)$

$\Rightarrow -10 + 4$

$\Rightarrow -6$

$-10 - (-4)$

$-10 + 4 = -6$

Kisi bhi Row/Column Expand

$|A| = 15$

$A_{3\times3} = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 2 \\ 3 & 6 & 0 \end{bmatrix} \Rightarrow$

C_1 (⊕)	C_2 (⊖)	C_3 (⊕)	
1	4	-1	$\Rightarrow R_1$
2	5	2	$\Rightarrow R_2$
3	6	0	$\Rightarrow R_3$

$1\begin{vmatrix} 5 & 2 \\ 6 & 0 \end{vmatrix} - 2\begin{vmatrix} 4 & -1 \\ 6 & 0 \end{vmatrix} + 3\begin{vmatrix} 4 & -1 \\ 5 & 2 \end{vmatrix}$

$= 1(0 - 12) - 2(0 - (-6)) + 3(8 - (-5))$

$= -12 - 2(6) + 3(13)$

$= -12 - 12 + 39$

$= -24 + 39$

$= -24 + 39$

$= 15$ Ans

$1(0 - 12) - 4(0 - 6) + (-1)(12 - 15)$

$-12 + 24 + 3$

15

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$$A^{-1} = \frac{1}{|A|}(Adj\,A)$$

Minors And Cofactors

$$A = \begin{bmatrix} 0 & 0 & -3 \\ 3 & -4 & 1 \\ 2 & 2 & 6 \end{bmatrix}$$

a_{11} a_{12} a_{13} a_{22} a_{23} a_{21} a_{31} a_{32} a_{33}

$$M_{11} = \begin{vmatrix} -4 & 1 \\ 2 & 6 \end{vmatrix} = -24 - 2 = -26$$

$$M_{12} = \begin{vmatrix} 3 & 1 \\ 2 & 6 \end{vmatrix} = 18 - 2 = 16$$

$$M_{13} = \begin{vmatrix} 3 & -4 \\ 2 & 2 \end{vmatrix} = 6 - (-8) = 14$$

$$M_{21} = \begin{vmatrix} 0 & -3 \\ 2 & 6 \end{vmatrix} = 0 - (-6) = 6$$

$$M_{22} = \begin{vmatrix} 1 & -3 \\ 2 & 6 \end{vmatrix} = 6 - (-6) = 12$$

$$M_{23} = \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$M_{31} = \begin{vmatrix} 0 & -3 \\ -4 & 1 \end{vmatrix} = 0 - (-12) = -12$$

$$M_{32} = \begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} = 1 - (-9) = 10$$

$$M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & -4 \end{vmatrix} = -4 - 0 = -4$$

$$C_{11} = -26 \quad C_{12} = -16 \quad C_{13} = 14$$

$$C_{21} = -6 \quad C_{22} = 12 \quad C_{23} = -2$$

$$C_{31} = -12 \quad C_{32} = -10 \quad C_{33} = -4$$

$$\text{Cofactor of } A = \begin{bmatrix} -26 & -16 & 14 \\ -6 & 12 & -2 \\ -12 & -10 & -4 \end{bmatrix}$$

$$\begin{bmatrix} -26 & -6 & -12 \\ -16 & 12 & -10 \\ 14 & -2 & -4 \end{bmatrix} = \text{Adjoint of } A = (Adj)A$$

$$A^{-1} = \frac{1}{68}\begin{bmatrix} -26 & -6 & -12 \\ -16 & 12 & -10 \\ 14 & -2 & -4 \end{bmatrix}$$

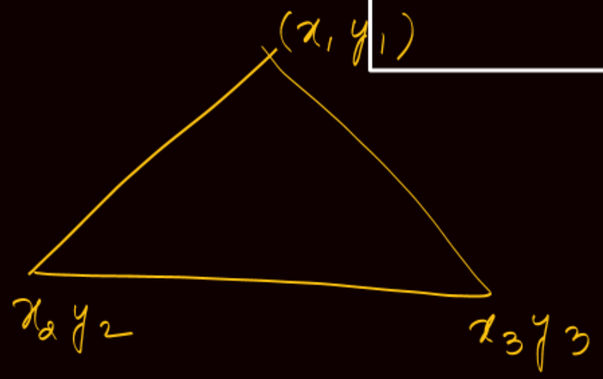
$-26 + 0 + 42 = -68$

A → M → [C]' → Adj.

$A^{-1} = \frac{1}{|A|}(Adj\,A)$

$$\text{Area of } \Delta = \left|\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

Collinear = $\Delta = 0$





Minors And Cofactors

$C_{ij} = (-1)^{i+j} M_{ij}$

A Determinant Associated With Each Element of a Sq. Matrix

$M_{11} = 6$ | $M_{12} = -3$ | $M_{21} = 4$ | $M_{22} = 2$

$C_{11} = 6$ | $C_{12} = 3$ | $C_{21} = -4$ | $C_{22} = 2$

Agar $i+j$ = Even, Minor = Cofactor

" $i+j$ = odd, Minor = - Cofactor

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -3 & 6 \end{bmatrix}$$

$M_{11} = 6$ $M_{12} = -3$

$M_{21} = 4$ $M_{22} = 2$

$C_{11} = (-1)^{1+1} M_{11} = 1 \times 6 = 6$

$C_{12} = (-1)^{1+2} M_{12} = -1 \times -3 = 3$

$C_{21} = (-1)^{2+1} M_{21} = -1 \times 4 = -4$

$C_{22} = (-1)^{2+2} M_{22} = 1 \times 2 = 2$

$$B = \begin{bmatrix} -2 & -3 \\ 4 & -9 \end{bmatrix}$$

$M_{21} = -3$ $M_{22} = -2$

$C_{21} = 3$ $C_{22} = -2$

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Inverse of Matrix

$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = |A|$

$a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0$

$|KA| = K^n|A|$ $\quad n \rightarrow$ order

$|A| = |A'|$ $\quad |AB| = |A||B|$

$AI = IA = A$ $\quad IA^{-1} = A^{-1}I = A^{-1}$

$AA^{-1} = A^{-1}A = I$

$(\text{Adj } A)A = A(\text{Adj } A) = |A|I$

$(\text{Adj } AB) = (\text{Adj } B)(\text{Adj } A)$

$|\text{Adj } A| = |A|^{n-1}$

$(AB)^{-1} = B^{-1}A^{-1}$

$AB = I$

B is Inverse of A

$B = A^{-1}$

$A^{-1} = \frac{1}{|A|}(\text{Adj } A)$

$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ formula

find A^{-1}

Solu : $A^2 - 4A + I$

Hence find A^{-1}

formula x

Dimag ✓

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$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Solving The Linear Equations → $A\vec{x} + B\vec{y} + C\vec{z} = D$

$AX = B$

\# $X = A^{-1}B \rightarrow X = \frac{1}{|A|}(\text{Adj}A)\,B$

$AX = O$ → Homogeneous Equations

x

$AX = B$ → Non Homogeneous Equations

$|A| \neq 0$ → Unique Sol^N ★

$|A| = 0$

- $(\text{Adj}A)\,B \neq 0$ → No Sol^N
- $(\text{Adj}A)\,B = 0$ → Many Sol^N



If $A=\begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}$, find the determinant of the matrix $A^2 - 2A$.

$= \begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$

$= \begin{vmatrix}5 & 0\\ 0 & 5\end{vmatrix} = 25 - 0$

$= 25$

$A^2 = \begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}$

$A^2 = \begin{bmatrix}7 & 6\\ 4 & 7\end{bmatrix}$

$2A = \begin{bmatrix}2 & 6\\ 4 & 2\end{bmatrix}$



If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$, find the values of x.

$2x(x-2) - (-3)(3x)$

$2x^2 - 4x + 9x = 3$

$2x^2 + 5x = 3$

$2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$

$2x(x+3) - 1(x+3) = 0$

$(x+3)(2x-1) = 0$

$x = -3$ $\quad x = \frac{1}{2}$



Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

$x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$-x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$

$-x^3 - x + x(\sin^2\theta + \cos^2\theta)$

$-x^3 - x + x$

$-x^3$



Prove that: .

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

$-a^2(b^2c^2 - b^2c^2) - ab(-abc^2 - abc^2) + ac(ab^2c + ab^2c)$

$-a^2 \times 0 \qquad -ab(-2abc^2) + ac(2ab^2c)$

$2a^2b^2c^2 + 2a^2b^2c^2$

$4a^2b^2c^2$



If the points (2, 3), (λ, - 1) and (0, 4) are collinear, find the value of λ.

Area of Δ : 0

$$\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ \lambda & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$$

$$2(-1-4) - 3(\lambda - 0) + 1(4\lambda) = 0$$

$$-10 - 3\lambda + 4\lambda = 0$$

$$\lambda = 10$$



Find the equation of the line joining A (1, 3) and B (0, 0) using determinants and find k if D (k, 0) is a point such that area of ABD is 3 sq. units.



Show that the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I_3 = 0$ and hence find A^{-1}.

→ Find A^{-1}

$$\frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$A^2 - 4A - 5I = 0$$

$$A^{-1}(A^2 - 4A - 5I) = A^{-1} \cdot 0$$

$$A^{-1} \cdot A \cdot A - 4A^{-1}A - 5A^{-1} \cdot I = 0$$

$$IA - 4I - 5A^{-1} = 0$$

$$A - 4I = 5A^{-1}$$

$$\frac{1}{5}(A - 4I) = A^{-1}$$

$A - 4I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$|A| = -2 + 5 + 1 = 4$

30-14-12



The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12. Using matrices find the numbers.

Let three no. be x, y, z

$x + y + z = 6$

$2z + x = 7$

$x + 0 \cdot y + 2z = 7$

$y + z + 3x = 12$

$3x + y + z = 12$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{|A|}(\text{Adj}A)B$$

$C_{11} = -2 \quad C_{12} = 5 \quad C_{13} = 1$

$C_{21} = 0 \quad C_{22} = -2 \quad C_{23} = +2$

$C_{31} = 2 \quad C_{32} = -1 \quad C_{33} = -1$

$$\text{Adj}A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} \begin{matrix} 3 \\ 1 \\ 2 \end{matrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$x = 3$

$y = 1$

$z = 2$

THANK YOU

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