LIVE

CLASS 12th APPLIED MATHS

BACKLOG MARATHON

SERIES

1. Boats and Streams
2. Races and Games
3. Alligation and Mixtures
4. Pipes and Cistern
5. Congruence Modulo and Modulo Arithmetic




WELCOME
TO CLASS
Next Toppers

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Unit 1 — 11 MARKS

Number Quantification & Numerical Application

Chapter 1

1. Modulo Arithmetic
2. Congruence Modulo.
3. Alligation & Mixture.
4. Boats & Streams.
5. Pipes & Cistern.
6. Race & Games.

Chapter 2

1. Numerical Inequalities

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Class 12th | Applied Maths

Alligation and Mixtures

Chapter 1


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शुद्धता हमारी पहचान
ग्राहक हमारी जान
आज का भाव
चावल - 60/-
दाल - 120/-
चना दाल - 110/-
मुंग दाल - 130/-
राजमा - 150/-
चना - 90/-
अच्छी क्वालिटी वाजिब दाम
चावल
₹ 60/
दाल
₹ 120/
चना दाल
₹ 110/-
गोलमाल है भाई,
सब गोलमाल है



Bhootiya Rule



Combined Arithmetic mean

or

weighted Arithmetic mean

$$\bar{x}_w = \frac{\sum wx}{\sum w}$$

$$\bar{x}_{12} = \frac{N_1 \times \bar{x}_1 + N_2 \times \bar{x}_2}{N_1 + N_2}$$

	I	II
P_1	30	40
Q_1	= 5 kg	2 kg

$$\text{mean price} = \frac{P_1 \times Q_1 + P_2 \times Q_2}{Q_1 + Q_2} \Rightarrow \frac{Q_1}{Q_2} =$$

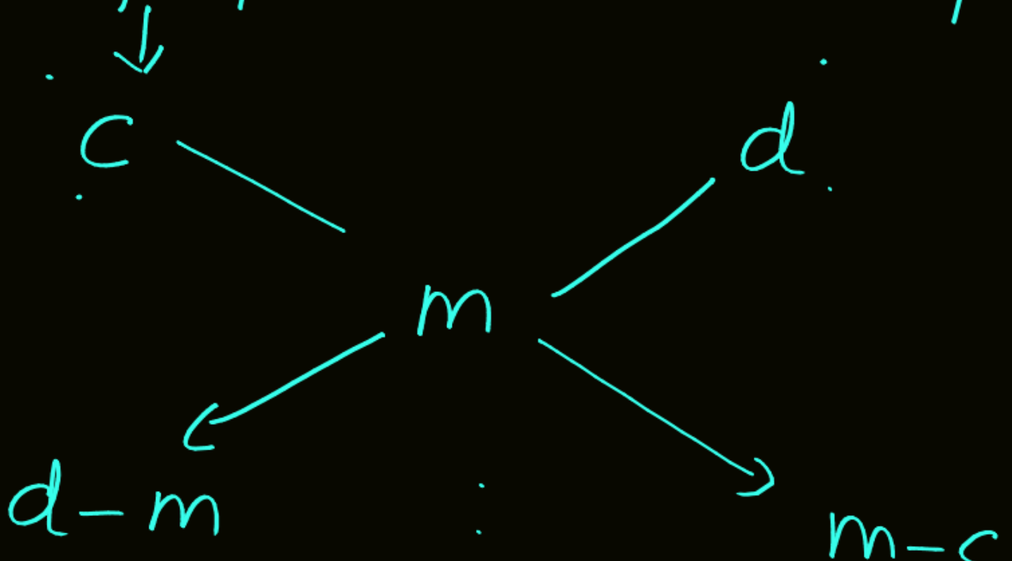
$$= \frac{30 \times 5 + 40 \times 2}{5 + 2}$$

$$=$$

Cheaper / I

Dearer / II

Cost Price



mixture
mean price

Prices should be of same type

$$\frac{d-m}{m-c} = \frac{\text{Qty of cheaper / I}}{\text{Qty of dearer / II}}$$



In what ratio must a shopkeeper mix two types of oranges worth ₹60 per kg and ₹90 per kg respectively so as to get a mixture at ₹80 per kg?

Given ⇒ C.P. of Type 1 oranges = ₹60
C.P of Type 2 oranges = 90 ₹
C.P. of mixture = ₹80

Ratio of Quantities ⇒ $\frac{Q_1}{Q_2} = ? = \frac{1}{2}$.

Ans: Hence, Both varieties of Oranges should be mixed in 1:2 ratio.

Using Alligation rule

Cheaper		dearer
60		90
	80	
90 − 80		80 − 60
10		20
= 1 : 2		

$$P\% = \frac{10}{100} = \frac{P}{CP}$$



In what ratio must a grocer mix two varieties of sugar worth ₹40 per kg and ₹50 per kg respectively so that by selling the mixture at ₹48.40 per kg, he may gain 10%?

Cheaper 40 — Dearer 50

Cost price — mixture — 44

Ratio of Qty = 3:2

50 − 44 : 44 − 40

6 : 4

= 3 : 2

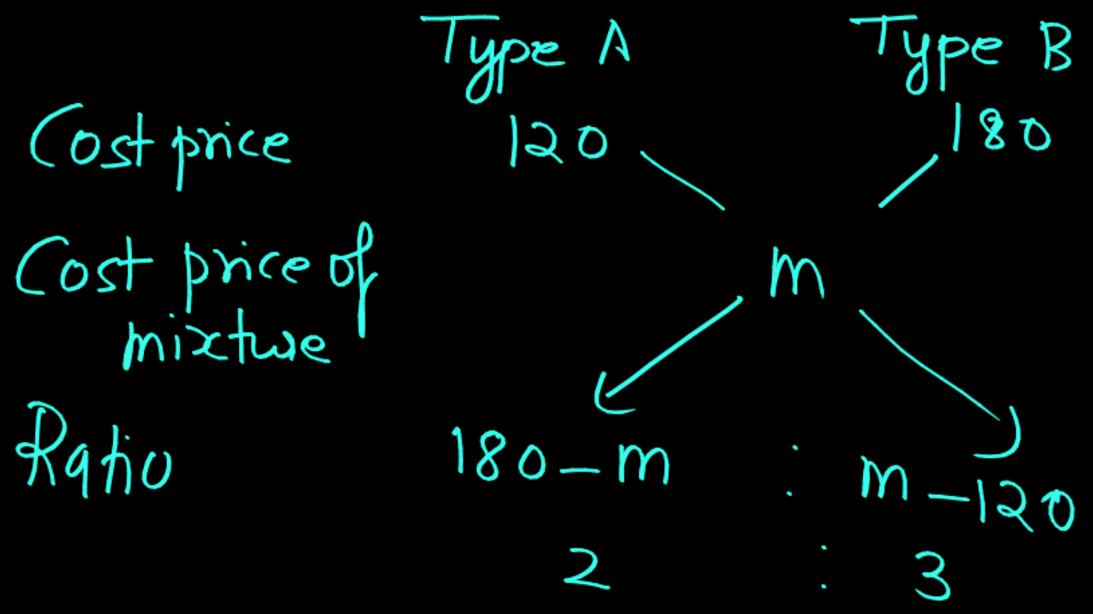
C	P	SP
100	10	110
x		48.40

$$\frac{48.40 \times 100}{110 \times 100} = x$$

$$= 44$$



The cost price of type A apples is ₹120 per kg and that of type B apples is ₹180 per kg. If both types of apples are mixed in the ratio 2 : 3 respectively, then find the price per kg of mixed apples. $\Rightarrow$ ₹156



$$\frac{180-m}{m-120} = \frac{2}{3}$$

$\Rightarrow (180-m)\times 3 = 2(m-120)$

$= 540 - 3m = 2m - 240$

$= 540 + 240 = 2m + 3m$

$= \frac{780}{5} = m$

$= ₹156 = m$



$C.P = 169$

$S.P = 202.80$

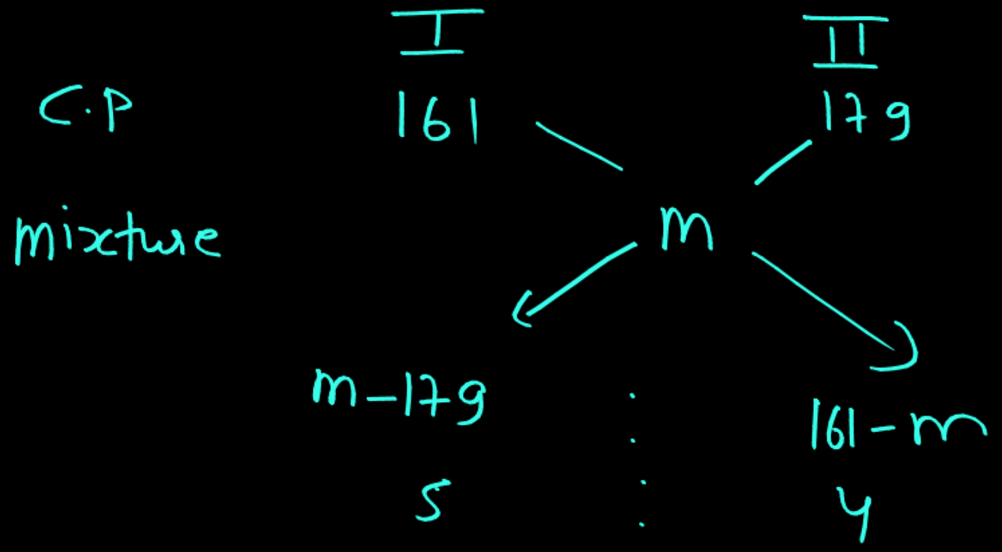
$P = S.P - C.P = 202.80 - 169$

$= 33.80$

$P\% = \frac{P}{C.P} \times 100 \Rightarrow \frac{33.80}{169} \times 100$ ~~260~~ 20%

20% = Profit

A shopkeeper mixes two varieties of rice one costs ₹161 per kg and other costs ₹179 per kg in the ratio 5 : 4 respectively. He sells the mixed variety at ₹202.80 per kg. Find his profit percentage.



$$\frac{m-179}{161-m} = \frac{5}{4}$$

$$4m - 716 = 805 - 5m$$

$$4m + 5m = 805 + 716$$

$$9m = 1521$$

$$m = \frac{1521}{9} = 169$$ ₹

$A : B = 3 : 2$

$\frac{392}{4} = 98$



Three varieties A, B and C of rice are mixed together in the ratio 4 : 1 : 1 respectively. The cost price of rice B is ₹82 per kg and that of rice C is ₹90 per kg. If the price of the mixture is ₹94 per kg, then find the price per kg of rice A.

	Price	Qty
Type A	P_1	4k
Type B	82	1k
Type C	90	1k
mixture	94	

$$\bar{x}_{123} = \text{average price} = \frac{P_1 \times Q_1 + P_2 \times Q_2 + P_3 Q_3}{Q_1 + Q_2 + Q_3}$$

$$94 = \frac{P_1 \times 4k + 82 \times 1k + 90 \times 1k}{4k + 1k + 1k}$$

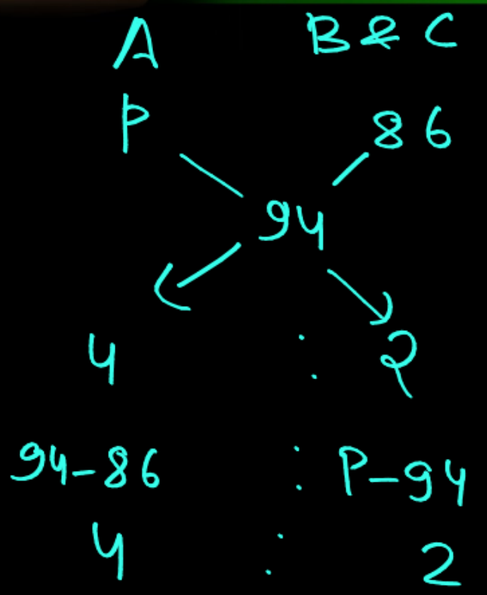
$$94 \times 6k = P_1 \times 4k + 82k + 90k$$

$$= \frac{564k - 172k}{4k} = P_1 \quad \therefore ₹98$$



2 : 3

Three varieties A, B and C of rice are mixed together in the ratio 4 : 1 : 1 respectively. The cost price of rice B is ₹82 per kg and that of rice C is ₹90 per kg. If the price of the mixture is ₹94 per kg, then find the price per kg of rice A.



A	B & C
P	$\frac{82\times1 + 90\times1}{1+1} = \frac{172}{2} = 86$

$$\frac{94-86}{P-94} = \frac{4}{2} \not\equiv 1$$

$$\Rightarrow 4 \not\equiv (P-94)\times 2$$

$$P = 98$$

$$\frac{82\times2 + 90\times3}{5}$$



900 g of sugar solution has 40% sugar in it. How much sugar should be added to make it 55% in the solution?

I (sugar soln) 40%

II sugar 100%

55%

100 − 55 : 55 − 40

45 : 15

3 : 1

$\frac{S.S}{S} = \frac{3}{1} = \frac{900}{x}$

$\Rightarrow x = \frac{900}{3}$

$= 300\ gm$



Let liquid X be 3k litres
& liquid Y be 5k litres

$16 \times \frac{5}{8} = Y$ $\quad x = 16 \times \frac{3}{8}$



A vessel contains a mixture of two liquids X and Y in the ratio 3 : 5. 8 litres of mixture are drawn off from the vessel and 8 litres of liquid X is filled in the vessel. If the ratio of liquids X and Y now becomes 7 : 10, how many litres of liquids X and Y were contained by the vessel initially?

$$\frac{X}{Y} = \frac{3k - \cancel{8} \times \frac{3}{\cancel{8}} + 8}{5k - 8 \times \frac{5}{8}} = \frac{7}{10}$$

$$\therefore \quad \frac{3k+5}{5k-5} = \frac{7}{10} \Rightarrow 30k + 50 = 35k - 35$$

$$85 = 5k$$

$$\frac{85}{5} = k = 17$$

$X = 3k = 3 \times 17$
$= 51$ Litres

$Y = 5k = 5 \times 17$
$= 85$ litres


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ग्राहक हमारी जान
आज का भाव
फुल क्रीम दूध - 60/-
टोंड दूध - 50/-
डबल टोंड दूध - 40/-
छाछ - 30/-
दही (500g) - 35/-
मलाई (100g) - 25/-
अच्छी क्वालिटी
वाजिब दाम
हमारा वादा
शुद्ध दूध
स्वस्थ परिवार
ताजा दूध
फुल क्रीम दूध
₹ 60 / लीटर
ताजा शुद्ध दूध
टोंड दूध
₹ 50 / लीटर



25%

A milkman has two cans. First containing 75% milk and rest water, whereas second containing 50% milk and rest water. How much mixture should he mix from each can so as to get 20 litres of mixture, such that ratio of milk and water is 5 : 3 respectively?

$\frac{5}{5+3} = \frac{5}{8}$

I

milk $75\% = \frac{75}{100} = \frac{3\times2}{4\times2} = \frac{6}{8}$

II

$50\% = \frac{50}{100} = \frac{1\times4}{2\times4} = \frac{4}{8}$

$\frac{5}{8}$

Ratio of Qty =) $\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$

$\frac{6}{8} - \frac{5}{8} = \frac{1}{8}$

Ratio = 1 : 1

Total mixture = 20L

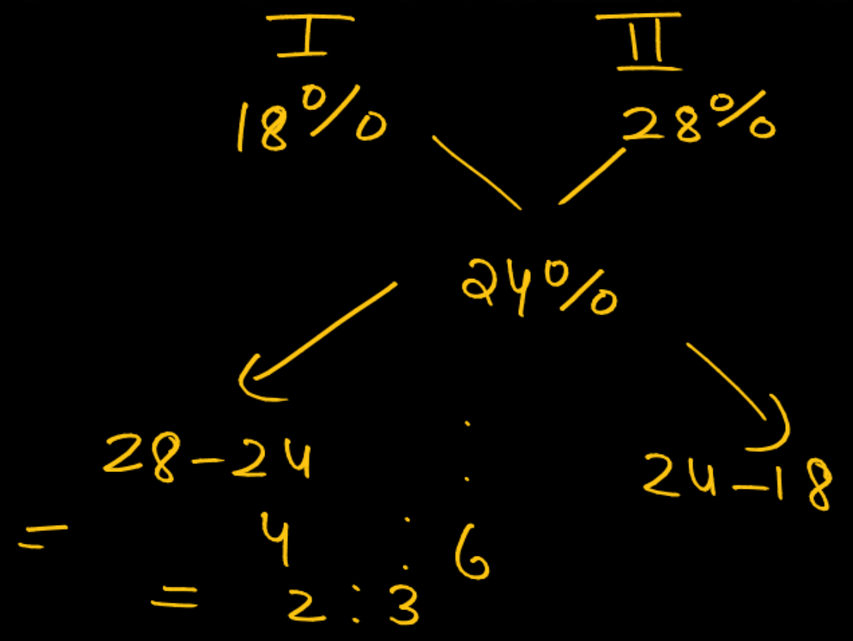
I = $20 \times \frac{1}{2} = 10$ L

II = $20 \times \frac{1}{2} = 10$ L



100 kg

A shopkeeper has 1 quintal of wheat, part of which he sells at 18% gain and the rest at 28% gain. In total she gains 24%. Find the quantity of wheat sold at 18% and 28%.



$I = 100 \times \frac{2}{5} = 40\text{ kg} \Rightarrow 18\%$

$II = 100 \times \frac{3}{5} = 60\text{ kg} \Rightarrow 28\%$



Challenge Time!!

In what ratio water must be added to dilute honey costing ₹240 per litre so that the resultant syrup would be worth ₹200 per litre?

Repeated Dilution



→ pure milk ⇒ 100 Litres

I ⇒ 10 Litre milk drawn & replaced by 10 L water

	M	W	Total
90% ←	100 − 10 = 90	10	100
	= 9 : 1		

II ⇒ 10 litres mixture was drawn & replaced by 10 L water

M ⇒ $90 - 10 \times \frac{9}{10}$

$= 90 - 9 = 81$

$90 \times \frac{90}{100}$

$= 81$ //

$10 - 10 \times \frac{1}{10}$

$= 10 - 1 = 9$ +10

$= 19$ L

100 =

Milk left at the end = $\left(100 \times \frac{90}{100}\right) \times \frac{90}{100}$

↓

Repeated Diluted

Milk left at the end = Original qty $\times \left(1 - \frac{\text{Replaced qty}}{\text{Original qty}}\right)^{n}$

$= x \times \left(1 - \frac{y}{x}\right)^{n}$



A container contains 50 litres of milk. From this container 10 litres of milk was taken out and replaced by water. This process was repeated two more times. How much milk is now left in the container?

$x = 50\ L$

$y = 10\ L$

$n = 3$

$$\text{Milk left} = x\left(1-\frac{y}{x}\right)^n$$

$$= 50\left(1-\frac{10}{50}\right)^3$$

$$= 50\left(\frac{4}{5}\right)^3$$

$$= 2\cancel{10}\ \cancel{\text{ltr}} \quad \frac{4}{\cancel{5}} \times \frac{4}{\cancel{5}} \times \frac{4}{5} = \frac{128}{5} = 25.6 \text{ litres}$$



From a container full of alcohol 6 litres of alcohol was drawn out and replaced by water. This process is repeated one more time. The ratio of the quantity of alcohol and water left in the container is 9 : 16. How many litres of alcohol did the container hold originally? x

$x = ?$

$y = 6$

$n = 2$

Alcohol $\Rightarrow x \times \frac{9}{25}$

Alcohol left $= x \times \left(1 - \frac{y}{x}\right)^n$

$x \times \frac{9}{25} = x \times \left(1 - \frac{6}{x}\right)^2$

$\Rightarrow \sqrt{\frac{9}{25}} = \frac{x-6}{x} = \frac{3}{5}$

$\Rightarrow 5x - 30 = 3x$

$\Rightarrow 5x - 3x = 30$

$2x = 30$

$x = 15$ Litres


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₹3499/-
₹2999/-
प्रारंभ 2.0
COMMERCE


Class 12th
₹4999/-
₹3999/-
प्रारंभ 2.0
PRO
COMMERCE+CUET

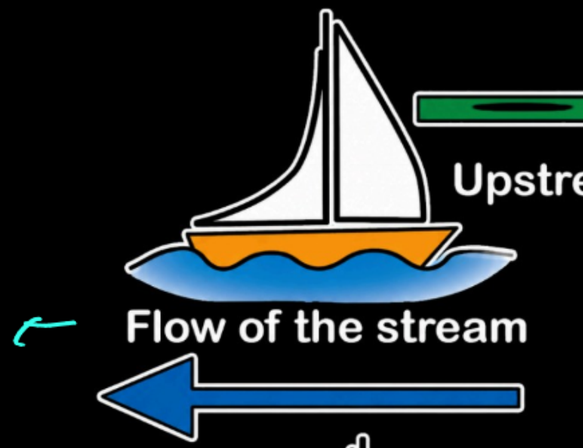
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Boats and Streams

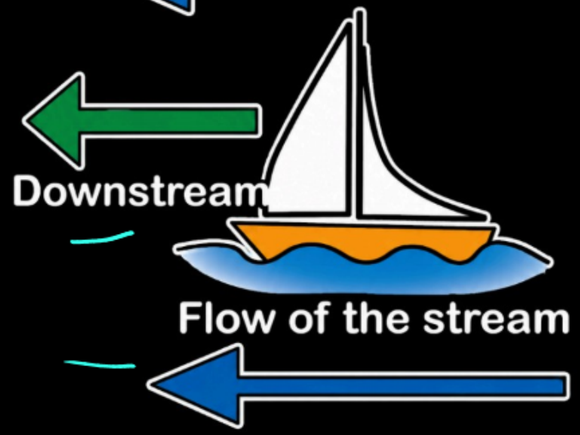


Upstream and Downstream



→ $(x-y)$ km/hr.

I $(x+y)$ →
Speed



Still water ⇒ Swimming pool ⇒ ruka hua pani

Stream / Current ⇒ flowing water

Speed of boat / person in still water = x km/hr

Speed of stream = y km/hr.

Speed in Downstream ⇒ $U = (x+y)$ km/hr.

Speed in upstream ⇒ $V = (x-y)$ km/hr.

I Given $\Rightarrow x$ & y

$u = x+y$

$v = x-y$

II Given $\rightarrow$ u & v

$x = \frac{u+v}{2}$

$y = \frac{u-v}{2}$

Speed = $\frac{\text{Distance}}{\text{Time}}$



A boat can row upstream at 10 km/h and downstream at 18 km/h. Find the speed of boat in still water and the speed of the stream.

$V = 10$ km/hr

$u = 18$ km/hr

$x = \frac{u+v}{2} = \frac{10+18}{2} = \frac{28}{2} = 14$ km/hr

$y = \frac{u-v}{2} = \frac{18-10}{2} = \frac{8}{2} = 4$ km/hr



A man rows 15 km upstream and 25 km downstream in 5 hours each time. What is the speed of the current? $\Rightarrow y = \frac{u-v}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$ km/hr

	Upstream	Downstream
D =	15 km	25 km
T =	5 hrs	5 hrs

$V = \frac{D}{T} = \frac{15}{5} = 3$ $\quad u = \frac{25}{5} = 5$



A woman can swim 8 km/h in still water. If the speed of the stream is 4 km/h, then find the time taken by the woman to cover the distance of 16 km upstream.

$x = 8$ km/hr

$y = 4$ km/hr

Upstream = 16 km

Speed of upstream = $v = 8 - 4$

$= 4$

$S = \frac{D}{T}$

$T = \frac{D}{S} = \frac{16}{4} = 4$ hrs //

$S = \frac{D}{T} \Rightarrow D = S \times T$ $\quad 5\frac{15}{60}\frac{1}{4} = \frac{21}{4}$



The ratio of speed of a motor boat in still water and that of the current of water is 25 : 4. The motor boat goes certain distance downstream in 5 hours 15 minutes. How much time will it take to come back?

$\frac{x}{y} = \frac{25k}{4k}$

Distance travelled D.S = Distance travelled U.S.

$S_u \times T_u = S_v \times T_v$

$29k \times \frac{21}{4} = 21k \times T_v$

$S_u = x + y = 25 + 4 = 29k$

$S_v = x - y = 25 - 4 = 21k$

$T_v = \frac{29}{4} = 7\frac{1}{4}$ hrs $\quad 7\text{hrs} + \frac{1}{4} \times 60^{15}$

$= 7\text{hrs } 15\text{min}$ //

⟶ Downstream

A ——— 5hrs 15 min ——— B

Upstream ⟵

$$S = \frac{D}{T} \Rightarrow T = \frac{D}{S}$$



The speed of a boat in still water is 6 km/h. It takes twice as long to go upstream to a point as to return downstream to the starting point. What is the speed of the stream? $y = ?$

$x = 6$

$$T_v = 2 \times T_u$$

$$\frac{D}{S_v} = 2 \times \frac{D}{S_u}$$

$$\therefore \frac{1}{x-y} = 2 \times \frac{1}{x+y}$$

$$\therefore \frac{1}{6-y} = 2 \times \frac{1}{6+y}$$

$$6+y = 2(6-y)$$

$$\therefore 6+y = 12-2y$$

$$3y = 6$$

$$y = \frac{6}{3} = 2 \text{ km/h} //$$

$$D = S \times T$$



A man can swim a certain distance downstream in 3 hours and returns back the same distance upstream in 6 hours. If the speed of stream is 2 km/h, then find the speed of man in still water. $= x$

$y = 2$ km/hr

D.S $\Rightarrow$ 3 hrs

A $\longrightarrow$ B

6 hrs $\longleftarrow$ U.S

$$D_u = D_v$$

$$S_u \times T_u = S_v \times T_v$$

$$(x+y) \times 3 = (x-y) 6^2$$

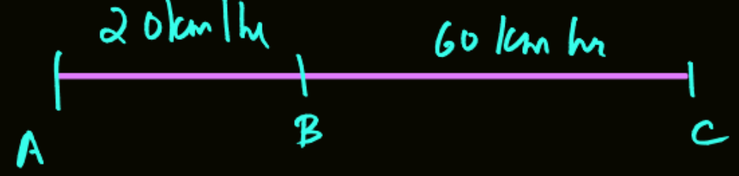
$$x+2 = (x-2) \times 2$$

$$x+2 = 2x-4$$

$$6 = x$$

km/hr //

Average Speed $\Rightarrow$ $\frac{S_1 + S_2}{2}$



$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken.}}$$

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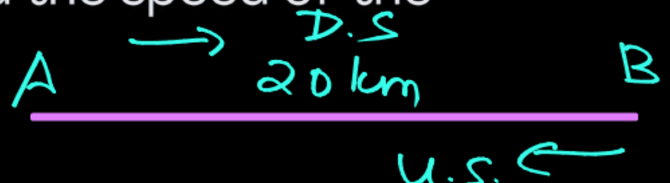
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$$S = \frac{D}{T} \Rightarrow T = \frac{D}{S}$$



Find the average speed of a boat in a round trip between two places 20 km apart. If the speed of boat in still water is 10 km/h and the speed of the stream is 5 km/h.

$x = 10$, $y = 5$



Average Speed = $\frac{\text{Total distance}}{\text{Total time}}$

$$T_u = \frac{D_u}{S_u} = \frac{D_4}{x+y} = \frac{20}{10+5} = \frac{20}{15} = \frac{4}{3}$$

$$T_v = \frac{D_v}{S_v} = \frac{20}{10-5} = \frac{20}{5} = 4$$

$$= \frac{20+20}{\frac{4}{3}+\frac{4\times3}{1\times3}} = \frac{40}{\frac{16}{3}} \Rightarrow \frac{40}{1}\times\frac{3}{16_2}$$

$$= \frac{15}{2} = 7.5 \text{ km/hr.}$$

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$$T = \frac{D}{S}$$



A motor boat can row at the speed of 8 km/h in still water. If the river is flowing at 4 km/h and it takes 16 hours for a round trip, find the distance between two places.

$x = 8$

$y = 4$

$T_u + T_v = 16$

$\frac{D}{S_u} + \frac{D}{S_v} = 16$

$\frac{D}{x+y} + \frac{D}{x-y} = 16$

$D\left[\frac{1}{8+4} + \frac{1}{8-4}\right] = 16$

$D\left[\frac{1}{12} + \frac{1\times3}{4\times3}\right] = 16$

$D \times \frac{4^{1}}{12} = \cancel{16}^{4}$

$D = 48\,km$ //



A swimmer whose speed in swimming pool is 4 km/h, swims between two points in a river and returns back to the starting point. He took 10 minutes more to cover the distance upstream than downstream. If the speed of the stream is 2 km/h, find the distance between two points.

$x = 4$ km/hr

$y = 2$ km/hr

$$T_V - T_u = \frac{10}{60}$$

$$= \frac{D}{x-y} - \frac{D}{x+y} = \frac{1}{6}$$

$$= D\left[\frac{1}{4-2} - \frac{1}{4+2}\right] = \frac{1}{6}$$

$$D\left[\frac{3\times1}{3\times2} - \frac{1}{6}\right] = \frac{1}{6}$$

$$D \times \frac{2}{6} = \frac{1}{6}$$

$$D = \frac{1}{2} = 0.5 \text{ km.}$$

$$= 500 \text{ m}$$ //

$$D = S \times T$$



A boat moving upstream takes 6 hours 36 minutes to cover a certain distance while it takes 3 hours to cover the same distance downstream.
Find the ratio of speed of boat in still water to the speed of stream. $x : y = ?$

A —(U.S.)→ B : 6 hrs 36 m

B ←(D.S.)— A : 3 hrs

$$D_u = D_v$$
$$S_u \times T_u = S_v \times T_v$$
$$(x+y) \times 3 = (x-y) \times \frac{33}{5}$$
$$5x + 5y = 11x - 11y$$
$$5y + 11y = 6x$$
$$16y = 6x$$

$$\frac{16}{6} = \frac{x}{y}$$
$$= \frac{8}{3} = \frac{x}{y}$$

$$6\,\frac{36}{60}\,\text{s}$$
$$= \frac{33}{5}$$



A boat goes 3.5 km upstream and then returns. Total time taken is 1 hour and 12 minutes. If the speed of the current is 1 km/h, then find the speed of the boat in still water.

Challenge Quiz

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Races and Games



RACE and GAME





Race

Race is a competition in which participants try to cover a fixed distance in the least possible time.



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Race course

A path on which a race takes place is called a race course
e.g. ground, road, swimming pool etc.





Starting point

A point from where the race starts is called the starting point.



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Finish point or Winning point or Goal

A point where race finishes is called the finish point or winning point or goal.





Winner

A person who first reaches the finish point is called winner.





Dead heat

If all the participants reach the finish point at the same instant of time, then this situation is called dead heat





A Beats B by 10 m in a Race Of 100 m





A Gives B a Start Of 10 m in a Race Of 100 m



$D_A = 100\ m$

$D_B = 100 - 10$

$= 90\ m$ //



A Gives B a Start Of 5 min

It is assumed that

A & B will reach
the finishing point
together until & unless
Ques specifies who reaches first.



A beats B by 10 m and 5 sec



Speed of B = $\frac{D}{T} = \frac{10}{5} = 2$ m/s.

Time of B = $\frac{D}{S} = \frac{100}{2} = 50$ sec.

Time of A = 50 - 5 = 45 sec



In a 600 m race, A beats B by 37.5 m or by 10 seconds, find the time taken by B to finish the race.

$$\text{Speed of B} = \frac{D}{T} = \frac{37.5}{10} = 3.75 \text{ m/s}$$

$$\text{Time of B to travel 600 m} = \frac{D}{S} = \frac{600}{37.5} \times 100 = 160 \text{ sec.}$$



In a 100 m race, A covers a distance in 36 seconds and B in 45 seconds. Find by how much distance A beats B?



A beats B by 9 sec.

D_B in 9 sec = ?

$$S_B = \frac{D}{T} = \frac{100}{45} = \frac{20}{9} \text{ m/s.}$$

Distance travelled by B in 9 sec = S × T

$$= \frac{20}{\cancel{9}} \times \cancel{9}$$

$$= 20 \text{ m} \quad //$$



A runs 3 times as fast as B. If A gives B a start of 50 m, find how far must be the finishing point from the starting point so that A and B reach the goal at the same time? ⇒ Race course = x m

$S_A = 3S_B$ $\quad D_A = x$

$\frac{S_A}{S_B} = \frac{3k}{1k}$ $\quad D_B = x - 50$

T_A to travel x m = T_B to travel (x − 50) m

$$\frac{x}{3k} = \frac{x-50}{1k}$$

$$x = 3x - 150$$

$$\frac{150}{2} = 75 \text{ m}$$ //

km/hr $\times\frac{5}{18}$ m/s



A and B participate in 200 m race. A runs at the speed of 6 km/h. A gives B a start of 18 m and still beats him by 6 seconds. Find the speed of B in km/h



$D_A = 200\ m$

$D_B = 200 - 18 = 182\ m$

$S_A = 6 \times \frac{5}{18} = \frac{5}{3}$ m/s

$T_A = \frac{D}{S} = \frac{200}{\frac{5}{3}} = 40 \times 3 = 120$ sec.

$S_B = \frac{D}{T} = \frac{182}{126} \times \frac{18}{5}$

$= \frac{26}{5} = 5.2$ km/h.

$T_B = 120 + 6 = 126$ sec.



In a 100 m race, A beats B by 20 m and C by 28 m. Find by how much distance B will beat C in a race of 500 m?

$$\frac{A}{B} = \frac{100}{100-20} = \frac{100}{80}$$

$$\frac{A}{C} = \frac{100}{100-28} = \frac{100}{72}$$

$$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C}$$

$$= \frac{100}{80} \times \frac{100}{72}$$

$$\frac{B}{C} = \frac{100 \times 5}{90 \times 5} = \frac{500}{450}$$

100 : 10
500 : x

$x = \frac{500 \times 10}{100} = 50$

In a 100 m race
B beats C by 10 m
Thus in a 500 m race
B beats C by $10 \times 5 = 50$ m



In a 200 m race, A can give a start of 18 m to B and a start of 31 m to C. In a race of 350 m, how much start can B give to C?

$$\frac{A}{B} = \frac{200}{200-18} = \frac{200}{182}$$

$$\frac{A}{C} = \frac{200}{200-31} = \frac{200}{169}$$

$$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C}$$

$$= \frac{\cancel{182}^{14}}{\cancel{200}} \times \frac{200}{\cancel{169}_{13}}$$

$$\frac{B}{C} = \frac{14 \times 25}{13 \times 25} = \frac{350}{325} = 25\text{m}$$

$$\begin{array}{cc} 14 & 1 \\ & \times \\ 350 & x \end{array}$$

$$x = \frac{350}{14}$$

$$= 25\text{ m}.$$



A can run 1 km in 4 minutes 54 seconds and B in 5 minutes. How many metres start can A give B in 1 km race so that the race may end in dead heat?

D_B in 6 sec.

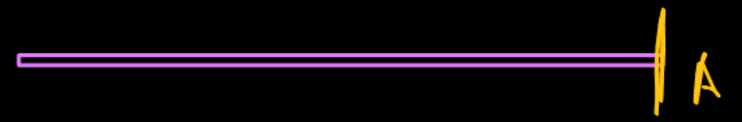
$$S_B = \frac{D}{T} = \frac{\cancel{1000}^{200}\,m}{\cancel{5} \times 60} = \frac{10}{3}\ m/s.$$

$$D = S \times T$$

$$= \frac{10}{\cancel{3}} \times \cancel{6}^{2} = 20\ m \ //$$



In a one kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.



A beats C by 45 sec. [30+15]
& by 180 m

Speed of C $= \frac{D}{T} = \frac{180}{45} = \frac{20}{5} = 4$ m/s

Time of C to travel 1km $= \frac{1000}{4} = 250$ sec.

$T_A = 250 - 45$
$= 205$ sec



imp

In a 500 m race, the ratio of the speeds of two participants A and B is 3:4 respectively. If A has a start of 140 m, then find the distance by which A wins.

$D_A = 500 - 140 = 360$ m $\quad s_A = 3$ m/s $\quad T_A = \frac{360}{3} = 120$ sec.

$D_B = 500$ m $\quad s_B = 4$ m/s. $\quad T_B = \frac{500}{4} = 125$ sec.

A beats B by 5 sec

D_B in 5 sec $= 5 \times 4 = 20$ m //

Game of 100 points

A = 100 points ✓

B = 80 points

A wins the game

↓

A beats B by 20 points

or

A can give B 20 points.



In a game of 100 points, A can give 20 points to B and 28 points to C. Find how many points can B give C. ⇒ 10 points.

$$\frac{A}{B} = \frac{100}{100-20} = \frac{100}{80}$$

$$\frac{A}{C} = \frac{100}{100-28} = \frac{100}{72}.$$

$$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C}$$

$$= \frac{10\cancel{80}}{\cancel{100}} \times \frac{\cancel{100}}{\cancel{72}_{9}}$$

$$\frac{B}{C} = \frac{100}{90}$$



imp ??

In a game, A can give 20 points to B, A can give 32 points to C and B can give 15 points to C. How many points make the game? = x

$$\frac{A}{B} = \frac{A}{C} \times \frac{C}{B}$$

$$\frac{A}{B} = \frac{x}{x-20}$$

$$\frac{A}{C} = \frac{x}{x-32}$$

$$\frac{B}{C} = \frac{x}{x-15}$$

$$\frac{x}{(x-20)} = \frac{x}{x-32} \times \frac{x-15}{x}$$

$$(x-32)x = (x-15)(x-20)$$

$$x^2 - 32x = x^2 - 15x - 20x + 300$$

$$35x - 32x = 300$$

$$3x = 300$$

$$x = 100 \text{ points}$$ //



imp

In a game of 90 points, A can give 15 points to B and 30 points to C. How many points can B give C in a game of 100 points?

$$\frac{A}{B} = \frac{90}{90-15} = \frac{90}{75}$$

$$\frac{A}{C} = \frac{90}{90-30} = \frac{90}{60}$$

$$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C}$$

$$\frac{B}{C} = \frac{\cancel{75}^{5}}{\cancel{90}} \times \frac{90}{60_{4}}$$

$$\frac{B}{C} = \frac{5 \times 20}{4 \times 20} = \frac{100}{80}$$

20 points

Class will Resume
at 7:35 pm

↓

After break => Pipes & Cistern

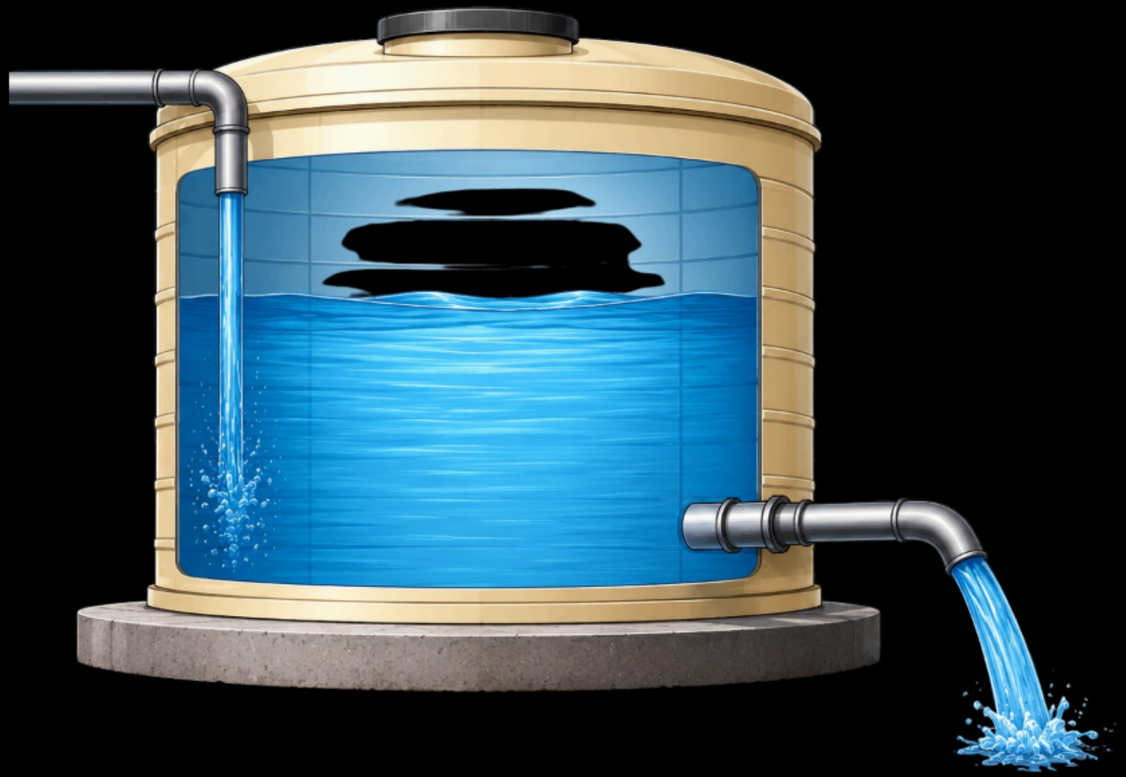
Get ready 

BACKLOG MARATHON SERIES

Class 12th | Applied Maths

Pipes & Cistern





Cistern ⇒ tank

inlet ⇒ pipe that fills the tank

outlet ⇒ that empties the tank / leakage.

HW = 50 Ques

↓

10 days

$\frac{50}{10}$ = 5 Ques every day

↓

consistency.

Pipe A = 5 hrs fills

Work to be done = 1

Part of tank Filled in hour = $\frac{1}{5} = \frac{1}{5}$ portion

$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

$= \frac{5}{5} = 1$

Time given

Work done in 1 hr $\Rightarrow \frac{1}{T_A}$

Work done in hr = given

Total time $= \frac{1}{\text{Work done in 1 hr.}}$

Part of tank filled in hr $= \frac{1}{7}$

Total time $= \frac{1}{\frac{1}{7}} = 7$ hrs //

Work done in 1 hr × Total hours = 1



Two pipes A and B can fill the tank in 20 minutes and 30 minutes respectively. If both the pipes are opened together, then find the time in which the tank will be filled completely.

Part of tank filled in 1 min by

$T_A = 20$ min $\quad A = \frac{1}{20} = \frac{1}{T_A}$

$T_B = 30$ min $\quad B = \frac{1}{30} = \frac{1}{T_B}$

$A \& B = \frac{3 \times 1}{3 \times 20} + \frac{1 \times 2}{30 \times 2}$

$A \& B = \frac{5}{60} \frac{1}{12}$

Thus $T_{AB} = \frac{1}{\frac{1}{12}}$

$= 12$ min //



A pipe can fill a tank in 20 minutes and another pipe can empty the same tank in 60 minutes. If both the pipes are opened together, then find how much time will they take to fill the tank?

$T_A = 20$ min $\quad \frac{1}{A} = \frac{1}{20}$

$T_B = 60$ min (empty) $\quad \frac{1}{B} = \frac{1}{60}$

$T_{AB} = 30$ min // $\quad \frac{1}{A} - \frac{1}{B} = \frac{1 \times 3}{20 \times 3} - \frac{1}{60} = \frac{2}{60}$ $\left(\frac{1}{30}\right)$



A pipe can fill a tank in 40 minutes. Due to a leakage in the bottom it took 60 minutes to fill the tank. How much time will it take for the leakage to empty the full tank?

$T_A = 40 \text{ min} \Rightarrow 1 \text{ min} \Rightarrow \frac{1}{A} = \frac{1}{40}$

$\text{leakage} = x \text{ min} \Rightarrow 1 \text{ min} = \frac{1}{x}$

$T_A \text{ \& leakage} = 60 \text{ min} \Rightarrow 1 \text{ min} = \frac{1}{60}$

$\frac{1}{A} - \frac{1}{x} = \frac{1}{60}$

$3x\,\frac{1}{3x\,40} - \frac{1_{x2}}{60x2} = \frac{1}{x}$

$= \frac{1}{120} = \frac{1}{x}$

$\Rightarrow x = 120 \text{ min}$ //



$x^2-35x+150=0$

$x^2-30x-5x+150=0$

$x(x-30)-5(x-30)=0$

$(x-30)(x-5)$

$x=30$ or $x=5$ ✗

Two pipes A and B together can fill a tank in 10 minutes. If pipe A takes 15 minutes less than B to fill the tank alone, then find the time taken by pipe B to fill the tank alone. $\Rightarrow$ 30 min //

$T_{AB}=10$ min $\Rightarrow$ 1 min $=\frac{1}{10}$

$T_B=(x)$ min $\Rightarrow$ 1 min $=\frac{1}{x}$

$T_A=(x-15)$ min $\Rightarrow$ 1 min $\Rightarrow\frac{1}{x-15}$

$$\frac{1}{A}+\frac{1}{B}=\frac{1}{10}$$

$$\frac{1}{x+15}+\frac{1}{x}=\frac{1}{10}$$

$$\frac{x+x-15}{(x-15)x}=\frac{1}{10}$$

$=(2x-15)\times 10=x^2-15x$

$=20x-150=x^2-15x$

$=x^2-35x+150=0$



Pipe A can fill the tank 2 times faster than pipe B. If both pipes A and B running together can fill the tank in 24 minutes, find how much time will pipe B alone take to fill the tank?

$S_A = 2 S_B$

$T_A = \frac{1}{2} T_B$, let $T_B = x$ min $= 1$ min $= \frac{1}{B} = \frac{1}{x}$

$T_A = \frac{x}{2}$

1 min $= \frac{1}{A} = \frac{1}{\frac{x}{2}} = \frac{2}{x}$

$T_{AB} = 24$ min

1 min $\frac{1}{A\&B} = \frac{1}{24}$

$\frac{1}{x} + \frac{2}{x} = \frac{1}{24}$

$= \frac{2+1}{x} = \frac{1}{24}$

$x = 24 \times 3 = 72$ min //.

20 Ques

$A = 4$ Ques $= T_A \Rightarrow \frac{20}{4} = 5$ days

$B = 2$ Ques $\Rightarrow T_B = \frac{20}{2} = 10$ days

$S_A = 2 S_B$

$\Rightarrow T_A = \frac{1}{2} T_B$



Two taps A and B can fill a tank in 4 and 6 hours respectively. The pipes are opened simultaneously and it takes 6 minutes more to fill the tank due to leakage. If the tank is full, then find the time taken by the leakage to empty the tank.

$T_A = 4$ hrs $\rightarrow$ 1 hr $= \frac{1}{4} = \frac{1}{A}$

$T_B = 6$ hrs $\rightarrow$ 1 hr $= \frac{1}{6} = \frac{1}{B}$

$T_{AB} \Rightarrow \frac{1}{A} + \frac{1}{B} = \frac{1}{3\times4} + \frac{1\times2}{6\times2} = \frac{5}{12}$

$T_{AB} = \frac{1}{\frac{5}{12}} = \frac{12}{5} = 2\text{hrs}\ \frac{2\times60}{5}^{12} = 2\text{hr } 24\text{ min}$ //

With leakage Time $\Rightarrow$ 2 hr 24 min + 6 min
$= 2$ hr 30 min $= 2.5$ hrs $= \frac{5}{2}$

$T_{A,B \& leakage} = \frac{5}{2}$ hrs

1 hr $= \frac{1}{\frac{5}{2}} = \frac{2}{5}$

$\frac{1}{A} + \frac{1}{B} - \frac{1}{x} = \frac{2}{5}$

$15\times\frac{1}{15\times4} + \frac{1\times10}{6\times10} - \frac{2\times12}{5\times12} = \frac{1}{x}$

$\frac{25-24}{60} = \frac{1}{x}$

$x = 60$ hrs //



A cistern can be filled by an inlet pipe in 20 hours and can be emptied by an outlet pipe in 25 hours. Both the pipes are opened. After 10 hours, the outlet pipe is closed, find the total time taken to fill the cistern.

$T_A = 20$ hrs $\Rightarrow 1$ hr $= \frac{1}{20}$

$T_B = 25$ hrs (empty) $\Rightarrow 1$ hr $= \frac{1}{25}$ (empty)

After 10 hrs, let inlet pipe be opened for x hrs.

Total time $= 10 + x = 10 + 18$

$= 28$ hrs

$\left(\frac{5\times1}{5\times20} - \frac{1\times4}{25\times4}\right) \times 10 + \frac{1}{20} \times x = 1$

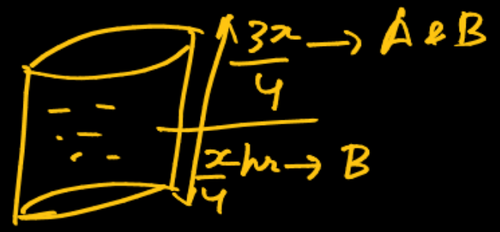
$= \frac{1}{10} \times 10 + \frac{1}{20} x = 1$

$= \frac{x}{20} = 1 - \frac{1}{10}$

$= \frac{x}{20} = \frac{9}{10}$

$= x = 18$ hrs.





Two pipes A and B can fill a tank in 20 hours and 16 hours respectively. Pipe B alone is kept open for ¼ th of the time and both pipes are kept open for the remaining time. Find the time required to fill the tank.

$T_A = 20$ hrs $\rightarrow 1$ hr $\rightarrow \frac{1}{20} = \frac{1}{A}$

$T_B = 16$ hrs $\rightarrow 1$ hr $\rightarrow \frac{1}{16} = \frac{1}{B}$

$$\left(\frac{1}{B}\right)\times\frac{x}{4} + \left(\frac{1}{A}+\frac{1}{B}\right)\frac{3x}{4} = 1$$

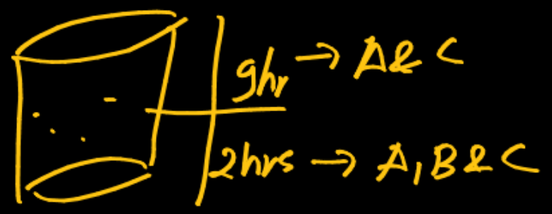
$$\frac{1}{16}\times\frac{x}{4} + \left(\frac{4\times1}{4\times20} + \frac{1\times1}{16\times1}\right)\times\frac{3x}{4} = 1$$

$$= \frac{5\times x}{5\times 64} + \left(\frac{9}{80}\right)\times\frac{3x}{4} = 1$$

$$= \frac{5x+27x}{320} = 1 \Rightarrow \frac{32x}{320} = 1$$

$x = 10$ hrs //





Three pipes A, B and C can fill a tank together in 8 hours. After working together for 2 hours B is closed and A and C can fill the remaining part in 9 hours. Find the time in which B alone can fill the tank.

$T_{ABC} = 8\ hrs \Rightarrow 1\ hr = \overbrace{\frac{1}{A}+\frac{1}{B}+\frac{1}{C}} = \frac{1}{8}$

$\frac{1}{12}+\frac{1}{B}=\frac{1}{8}$

$\frac{1}{B} = \frac{3\times1}{3\times8} - \frac{1\times2}{12\times2}$

$\frac{1}{B} = \frac{1}{24}$

$B = 24\ hrs$ //

$\left(\frac{1}{A}+\frac{1}{B}+\frac{1}{C}\right)\times2 + \left(\frac{1}{A}+\frac{1}{C}\right)\times9 = 1$

$= \frac{1}{8}\times2 + \left(\frac{1}{A}+\frac{1}{C}\right)\times9 = 1$

$= \left(\frac{1}{A}+\frac{1}{C}\right)\times9 = 1-\frac{1}{4}$

$= \frac{1}{A}+\frac{1}{C} = \frac{3}{4\times9\times3} = \frac{1}{12}$



1 min = 5 L

24 min ⇒ 24 × 5 = 120 Litres = Capacity //

Two pipes P and Q can fill a tank in 60 minutes and 30 minutes respectively. Third pipe R drains 5 litres of water per minute. If all the three pipes are opened it took 2 hours to fill the tank. Find the capacity of the tank.

T_P = 60 min ⇒ 1 min = $\frac{1}{P} = \frac{1}{60}$

T_Q = 30 min ⇒ 1 min = $\frac{1}{Q} = \frac{1}{30}$

T_R = ? ⇒ x min ⇒ 1 min = $\frac{1}{x}$ (empty)

T_{PQR} ⇒ 2 hrs = 120 min ⇒ 1 min = $\frac{1}{120}$

$$\frac{1}{P} + \frac{1}{Q} - \frac{1}{R} = \frac{1}{120}$$

$$\frac{1}{60} + \frac{1}{30} - \frac{1}{x} = \frac{1}{120}$$

$$\frac{2x \cdot 1}{2x \cdot 60} + \frac{4x \cdot 1}{4x \cdot 30} - \frac{1}{120} = \frac{1}{x}$$

$$\frac{1}{120_{24}} = \frac{1}{x} \Rightarrow x = 24 \text{ min.}$$



$T_A = 6$ hrs
$T_B = 4$ hrs
$T_C = 12$ hrs //

Three pipes A, B and C can fill a tank together in 2 hours. Pipes B and C take 3 hours to fill the tank, while pipes A and C take 4 hours to fill the tank. Find the time taken by each pipe to fill the tank alone.

$T_{ABC} = 2\text{hrs} \Rightarrow 1\text{hr} \Rightarrow \frac{1}{A} + \frac{1}{B} + \frac{1}{C} = \frac{1}{2}$ - ①

$T_{BC} = 3\text{hrs} \Rightarrow 1\text{hr} \rightarrow \frac{1}{B} + \frac{1}{C} = \frac{1}{3}$ - ②

$T_{AC} = 4\text{hrs} \Rightarrow 1\text{hr} \Rightarrow \frac{1}{A} + \frac{1}{C} = \frac{1}{4}$ - ③

for A ⇒ From eq ① & ②

$\frac{1}{A} + \frac{1}{3} = \frac{1}{2}$

$\frac{1}{A} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} = \frac{1}{A}$

for B ⇒ from eq ① & ③

$\frac{1}{4} + \frac{1}{B} = \frac{1}{2}$

$\frac{1}{B} = \frac{2\times1}{2\times2} - \frac{1}{4} = \frac{1}{4}$

$\frac{1}{A} = \frac{1}{6}$, $\frac{1}{B} = \frac{1}{4}$

Put in eq ①

$\frac{1}{6} + \frac{1}{4} + \frac{1}{C} = \frac{1}{2}$

$\frac{1}{C} = \frac{1\times6}{2\times6} \quad \frac{1\times2}{6\times2} \quad \frac{1\times3}{4\times3}$

$\frac{1}{C} = \frac{1}{12}$

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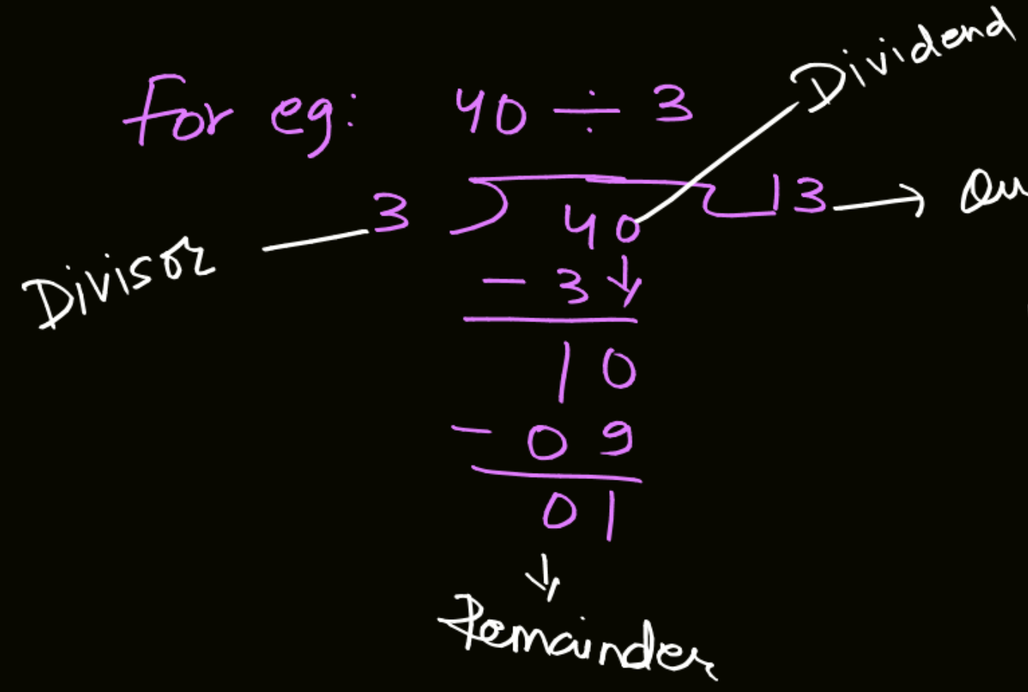
Congruence & Arithmetic Modulo

Remainder Theorem

$r = 0, 1, 2, \cancel{3}$

Dividend = Divisor × Quotient + Remainder.

For eg: $40 \div 3$



$40 = 3 \times 13 + 01$

$40 = 39 + 01$

$0 \leq r < |\text{Divisor}|$

40 divided by 3 gives remainder 1

↓

modulo arithmetic

↓

$40 \bmod 3 = 1 \Rightarrow a \bmod b = r$

Eg: $36 \bmod 8 = 4$

$47 \bmod 5 = 2$

$a, b =$ integers

$b \neq 0$

$0 \leq r < |b|$



Introduction



Modulo arithmetic



Congruence Modulo

$a, b, n \Rightarrow$ integer

$n \neq 0$

$n \neq 1$

$18 \bmod 4 = 2$ $\qquad$ $22 \bmod 4 = 2$

$18 \equiv 22 \bmod 4$

$\because 1$

- 18 & 22 when divided by 4 gives same remainder
- diff. of 18 & 22 i.e $18-22=-4$ will by divisible by 4 //

$a \equiv b \bmod n$

- a & b leaves same remainder when divided by n
- a−b is the integral multiple of n i.e. a−b will be divisible by n.

eg: $49 \bmod 3 \Rightarrow 1$ $\qquad$ $57 \bmod 3 = 0$

$49 \equiv 57 \bmod 3$?

Properties of modulo arithmetic & Congruence modulo.

1. Addition properties

1. If $a+b = c$

$$(a+b) \bmod n \equiv c \bmod n$$

Or $$a \bmod n + b \bmod = c \bmod n$$

$$(53 + 72) \bmod 7 = 125 \bmod 7 = 6$$

$$53 \bmod 7 + 72 \bmod 7$$

$$4 \bmod 7 + 2 \bmod 7$$

$$(4+2) \bmod 7$$

$$6 \bmod 7$$

$$\Rightarrow 6$$

2) If $a \equiv b \bmod n$

$(a+k) \equiv (b+k) \bmod n$

eg: $23 \bmod 3 = 2$ $\quad$ $29 \bmod 3 = 2$

$23 \equiv 29 \bmod 3$

$(23+5) \equiv (29+5) \bmod 3$

$28 \equiv 34 \pmod 3$

$28 \to 1$ $\quad$ $34 \to 1$

③ If $a \equiv b \bmod n$ & $c \equiv d \bmod n$

then $(a+c) \equiv (b+d) \bmod n$

eg: $41 \equiv 66 \bmod 5$ $\quad$ $38 \equiv 23 \bmod 5$

$(41+38) \equiv (66+23) \bmod 5$

$79 \equiv 89$

$79 \to 4$ $\quad$ $89 \to 4$

Multiplication Properties.

1. If $a \times b = c$

   $a \times b \bmod n = c \bmod n$

   $\Rightarrow a \bmod n \times b \bmod n = c \bmod n$

2. If $a \equiv b \bmod n$

   then $(k \times a) \equiv (b \times k) \bmod n$

③ If $a \equiv b \bmod n$ & $c \equiv d \bmod n$

$(a \times c) \equiv (b \times d) \bmod n$

$2 \times 9 \equiv 13 \times 2 \bmod 4$

$18 \equiv 26$

$\downarrow \quad \downarrow$

$2 \quad 2$

③ $9 \equiv 13 \bmod 4$, $2 \equiv 6 \bmod 4$

$= 9 \times 2 \equiv 13 \times 6 \bmod 4$

$= 18 \equiv 78 \bmod 4$

$\downarrow \quad \downarrow$

$2 \quad 2$

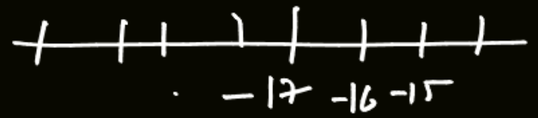


Evaluate the following:

(i) $43 \pmod 6$ (ii) $97 \pmod 5$ (iii) $-17 \pmod 4$

$= 1$

2



17 mod 4	−17 mod 4	17 mod (−4)	−17 mod (−4)
$17 = 4 \times 4 + 1$ $4 \overline{)17}\ 4$ -16 1	$-17 = 4 \times (-5) + 3$ $4 \overline{)-17}\ -5$ $-(-20)$ $-17 + 20 = 3$ $-17 = 4 \times (-5) + 3$ $-17 = -20 + 3$ $-17 = -17$ //	$17 = -4 \times (-4) + 1$ $-4 \overline{)17}\ -4$ -16 1 $17 = -4 \times (-4) + 1$ $= 16 + 1$ $= 17$	$-17 = -4 \times 5 + 3$ $-4 \overline{)-17}\ 5$ $-(-20)$ $-17 + 20 = +3$ $5 \times (-4) + 3$ $-20 + 3$ $= -17$ //



Find (576+789) mod 9.

$$576 \bmod 9 + 789 \bmod 9$$

$$= 0 \bmod 9 + 6 \bmod 9$$

$$= (0+6) \bmod 9$$

$$= 6 \bmod n$$

↓

Remainder //



Find the remainder when $(672 + 541 + 383 + 295 + 101 + 86)$ is divided by 3.

$$(672 + 541 + 383 + 295 + 101 + 86) \bmod 3$$

$$\Rightarrow 672 \bmod 3 + 541 \bmod 3 + 383 \bmod 3 + 295 \bmod 3 + 101 \bmod 3 + 86 \bmod 3$$

$$= 0 \bmod 3 + 1 \bmod 3 + 2 \bmod 3 + 1 \bmod 3 + 2 \bmod 3 + 2 \bmod 3$$

$$= (0 + 1 + 2 + 1 + 2 + 2) \bmod 3$$

$$= 8 \bmod 3$$

$$= 2 \bmod 3$$

$\therefore$ R = 2



Find the remainder when 783 × 657 × 594 × 432 × 346 × 251 is divided by 5.

$783 \bmod 5 \times 657 \bmod 5 \times 594 \bmod 5 \times 432 \bmod 5 \times 346 \bmod 5 \times 251 \bmod 5$

$= 3 \bmod 5 \times 2 \bmod 5 \times 4 \bmod 5 \times 2 \bmod 5 \times 1 \bmod 5 \times 1 \bmod 5$

$= (3 \times 2 \times 4 \times 2 \times 1 \times 1) \bmod 5$

$= 48 \bmod 5$

$= 3 \bmod 5$

$\therefore R$



Find the last two digits of the product 4895 × 6789.

$$(4895 \times 6789) \bmod 100$$

$$\Rightarrow 4895 \bmod 100 \times 6789 \bmod 100$$

$$= 95 \bmod 100 \times 89 \bmod 100$$

$$= (95 \times 89) \bmod 100$$

$$(8455) \bmod 100$$

$$= 55 \bmod 100$$

↓ last 2 digits

4
4
95
× 89
855
760×
8455



a & b ⇒ 2 positive integers

m ⇒ modulo

Properties

Addition modulo	Subtraction modulo	Multiplication modulo
$a +_m b =$ I $a+b \geq m$ $= (a+b) \bmod m$ II $a+b < m$ $= a+b$	$a -_m b =$ I $a-b \geq m$ $= (a-b) \bmod m$ II $a-b < m$ $\Rightarrow a-b$ //	$a \times_m b =$ I $a \times b \geq m$ $\Rightarrow a \times b \bmod m$ II $a \times b < m$ $\Rightarrow a \times b$



(i) Find addition modulo 7 if a and b are 12 and 6 respectively.

$m = 7$

$a = 12$

$b = 6$

$a + b = 12 + 6 = 18 > 7$

$18 \bmod 7$

$= 4$ //



(ii) Find addition modulo 19 if a and b are 8 and 6 respectively.

$m = 19$

$a = 8$

$b = 6$

$a + b = 14 < 19$

$r = 14$ //



(i) Find subtraction modulo 8 if a and b are 15 and 4 respectively.

$$15 - 4 = 11 \bmod 8$$
$$= 3$$



(ii) Find subtraction modulo 12 if a and b are 20 and 12 respectively.

$20 - 12$

$= 8 < 12$

↓

remainder.



(i) Find multiplication modulo 8 if a and b are 5 and 7 respectively.

$7 \times 5 = 35 > 8$

$35 \bmod 8$

$= 3$



(ii) Find multiplication modulo 6 if a and b are 2 and 3 respectively.

Challenge Ques

Properties of Exponents.

If $a \equiv b \mod n$

then $a^k \equiv b^k \mod n$

eg: $4 \equiv 7 \mod 3$

$4^2 \equiv 7^2 \mod 3$

$16 \equiv 49 \mod 3$

$16 \downarrow 1$

$49 \downarrow 1$



3 ki kitni power raise karu so that jo no. aaye us me se (1) ya (−1) subtract karne par jo no. mile wo 4 se divisible ho

Find $3^{16} \pmod 4$. $= r$

$3^1 \equiv (-1)^1 \bmod 4$

$3^{1\times16} \equiv (-1)^{16} \bmod 4$

$3^{16} \equiv 1 \bmod 4$

$\therefore 3^{16} \bmod 4 = 1$ //

$3^1 - 1 = 2 \div 4$ ✗

$3^1 - (-1) = 4 \div 4$ ✓

$a \equiv b \bmod 4$



Find the last digit of 12^{12}. 6

$12^{12} \bmod 10 = r$

$12 \equiv 2 \bmod 10$

$12^6 \equiv 2^6 \bmod 10$

$12^6 \equiv 64 \bmod 10$

$12^6 \equiv 4 \bmod 10$

$= 12^{6\times 2} \equiv 4^2 \bmod 10$

$= 12^{12} \equiv 16 \bmod 10$

$12^{12} \equiv 6 \bmod 10$

$12^1 \Rightarrow 12-1 = 11 \div 10$ ✗

$12^1 \Rightarrow 12-(-1) = 13 \div 10$ ✗

$12^1 \Rightarrow 12-2 = 10 \div 10$ ✓

a b



Find the remainder when 5^{61} is divided by 7.

$5^{61} \bmod 7 = r$

$5^1 = 5-1 = 4 \div 7$ ✗

$5^1 = 5-(-1) = 6 \div 7$

$5^3 = 125-(-1) = 126 \div 7$

$=$ ✓

$5^3 \equiv (-1) \bmod 7$

$5^{3\times 20} \equiv (-1)^{20} \bmod 7$

$= 5^{60} \equiv 1 \bmod 7$

$= 5^{60} \times 5^1 \equiv 1 \times 5 \bmod 7$

$= 5^{61} \equiv 5 \bmod 7$

$= 5$



Find the remainder when 5^{61} is divided by 7.



Find the value of $6^{12} (\mathrm{mod} 7)$.

Challenge Que 3



Find the units digit in 7^{295}.

$\frac{295}{2} = 147$

$7^{295} \bmod 10$

$7^2 = 49 - (-1) = 50 \div 10$ ✓

$7^2 \to a$, $(-1) \to b$

$7^2 \equiv (-1) \bmod 10$

$7^{2\times147} \equiv (-1)^{147} \bmod 10$

$7^{294} \equiv (-1) \bmod 10$

$7^{294} \times 7^1 \equiv (-1) \times 7 \bmod 10$

$= 7^{295} \equiv (-7) \bmod 10$

$= 7^{295} \equiv 3 \bmod 10$ //

last digit //

$18 \div 3 = 0$

$19 \div 3 = 1$

$20 \div 3 = 2$

$21 \div 3 = 0$

24 hrs = 10pm

+6 hrs

= 4 am

$30 \bmod 24 \Rightarrow 6$



490 hrs

It is 7:00 P.M. currently. What time (in A.M. or P.M.) will be in next 1500 hours?

+12 hrs ⇒ 7 a.m

1500 mod 24 =

24) 1500 (62

144 ↓

60

48

12

7.00 pm

+10 hrs

= 5 a.m

Equivalence Class
or Congruence Class.

$$[a]_n$$

set of those values which are congruent to $a \bmod n$

$$[13]_5$$

-5 -5 $+5$ $+5$ $+5$ $+5$

3 8 13 18 23 28

$$13 \pm 5 \pm 10 \pm 15$$



If $39 \equiv x \pmod 7$, find the set of values of x.

$39 - x = 7k$

$39 - 7k = x$

$[39]_7$

$k = 0 = 39 - 0 = 39$

$k = 1 = 39 - 7 = 32$

$-2 \quad -7 \quad \leftarrow \quad 39 \quad \xrightarrow{+7} \quad +2$

$39, \pm 7, \pm 14, \pm 21 \ldots$



Find the set of values of x satisfying $x + 1 \equiv 32 \pmod 7$.

$x + 1 - 32 = 7k$

$x - 31 = 7k$

$x = 7k + 31$



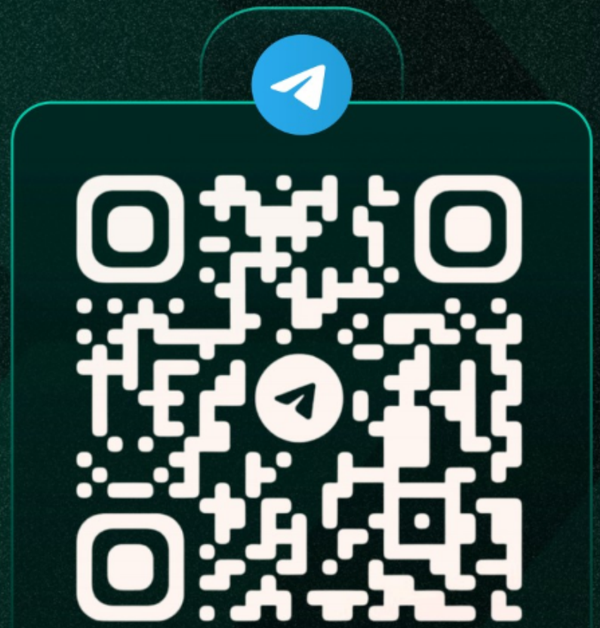
Find the set of values of x satisfying $x \equiv 12 \pmod 5$.

Challenge Que 4.

THANK YOU

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