

Class 12th

word Problem

Application of Derivatives

The surface area of a cube increases at the rate of $72 \text{ cm}^2/\text{sec}$. Find the rate of change of its volume when the edge of the cube measures 3 cm.

Let Edge of Cube = $x \text{ cm}$.

Surface Area of Cube inc. = $72 \text{ cm}^2/\text{s}$

$$S = 6a^2$$

$$\frac{ds}{dt} = 6 \times 2a \times \frac{da}{dt}$$

$$72 = 6 \times 2 \times a \times \frac{da}{dt}$$

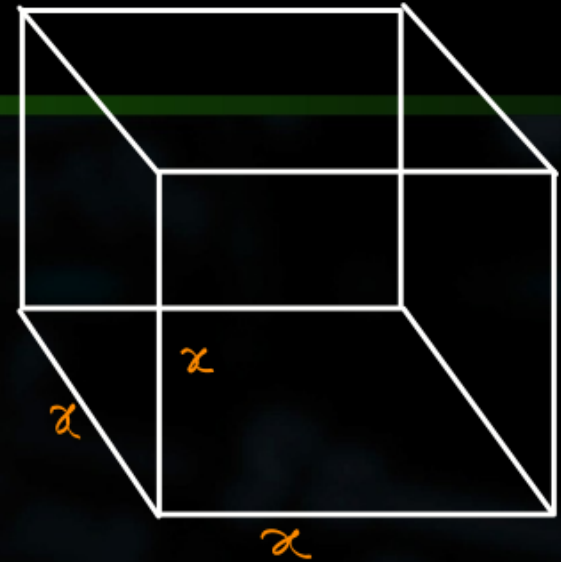
$$\frac{6}{a} = \frac{da}{dt}$$

$$V = a^3$$

$$\frac{dv}{dt} = 3a^2 \times \frac{da}{dt}$$

$$= 3a^2 \times \frac{6}{a} = 18a$$

$$\left. \frac{dv}{dt} \right|_{a=3} = 18 \times 3 = 54 \text{ cm}^3/\text{s}$$



Sand is pouring from a pipe at the rate of $15 \text{ cm}^3/\text{min}$. The falling sand forms a cone on the ground such that the height of the cone is always one third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4 cm ?

$\frac{dh}{dt} = ?$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (3h)^2 h$$

$$V = \frac{1}{3} \pi \times 9h^2 \times h$$

$$V = 3\pi h^3$$

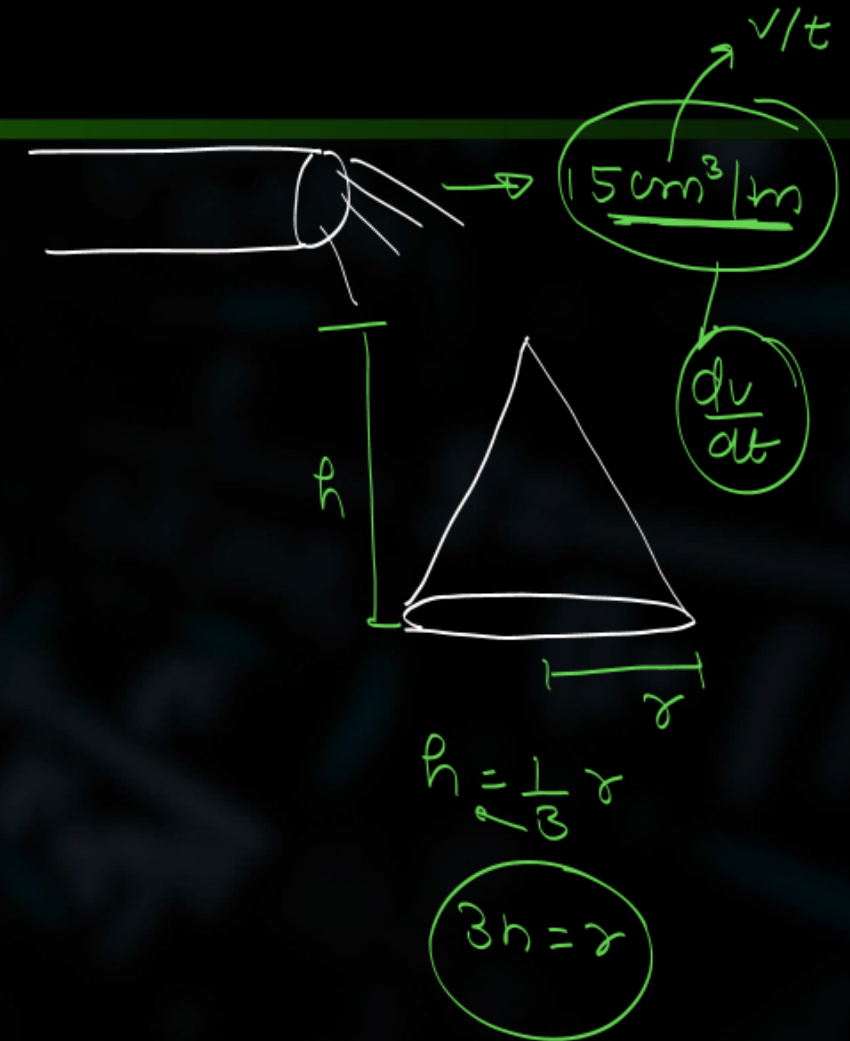
$$\frac{dV}{dt} = 3\pi \times 3h^2 \times \frac{dh}{dt}$$

$$15 = 9\pi h^2 \times \frac{dh}{dt}$$

$$\frac{5}{39\pi h^2} = \frac{dh}{dt}$$

$$\frac{5}{3\pi(4)^2} = \frac{dh}{dt} \bigg|_{h=4}$$

$$\frac{5}{48\pi} \text{ cm/min} = \frac{dh}{dt}$$



C.P.
 $f'(x) < 0 \rightarrow \text{Max.}$
 $f'(x) > 0 \rightarrow \text{Min.}$

Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

$y = 5 - x$
 $x + y = 5$

$x^3 + y^3 = \text{least}$ Min.

$x^2 + y^2 = ?$

$\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2$
 $\frac{25}{4} + \frac{25}{4} = \frac{50}{4} = \frac{25}{2}$ Ans.

$f(x) = x^3 + (5-x)^3$

$f'(x) = 3x^2 + 3(5-x)^2 \times (0-1)$

$f'(x) = 3x^2 - 3(5-x)^2$

$f'(x) = 0$

$3x^2 - 3(5-x)^2 = 0$

$3[x^2 - (5-x)^2] = 0$

$x^2 - (25 + x^2 - 10x) = 0$

$x^2 - 25 - x^2 + 10x = 0$

$10x = 25$
 $x = \frac{25}{10} = \frac{5}{2}$

$x = \frac{5}{2}$

$y = 5 - \frac{5}{2}$

$y = \frac{5}{2}$

$f'(x) = 3x^2 - 3(5-x)^2$

$f'(x) = 6x - 3[2(5-x)(0-1)]$

$= 6x + 3[10 - 2x]$

$= 6x + 30 - 6x = 30 > 0$

$f''(x) > 0$

Hence $f(x)$ is Min

$$V \rightarrow \text{const.}$$

$$V = x \times x \times h \Rightarrow V = x^2 \times h \Rightarrow \frac{V}{x^2} = h$$

$$h = \frac{x}{2}$$

An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when the depth of the tank is half of its width.

length of cube = x , width = x , height = h

S.A of Metal Used = C.S.A + B.A

$$= 2h(x+x) + x^2$$

$$= 2h \times 2x + x^2$$

$$= 4hx + x^2$$

$$f(x) = 4x \times \frac{V}{x^2} + x^2$$

$$f(x) = \frac{4V}{x} + x^2$$

$$f'(x) = 4V \left(-\frac{1}{x^2} \right) + 2x$$

$$-\frac{4V}{x^2} + 2x = 0$$

$$2x = \frac{4V}{x^2}$$

$$2x^3 = 4 \times \frac{x^2 h}{x^2}$$

$$\frac{x}{2} = h$$

$$x^{-2} = -2x^{-3}$$

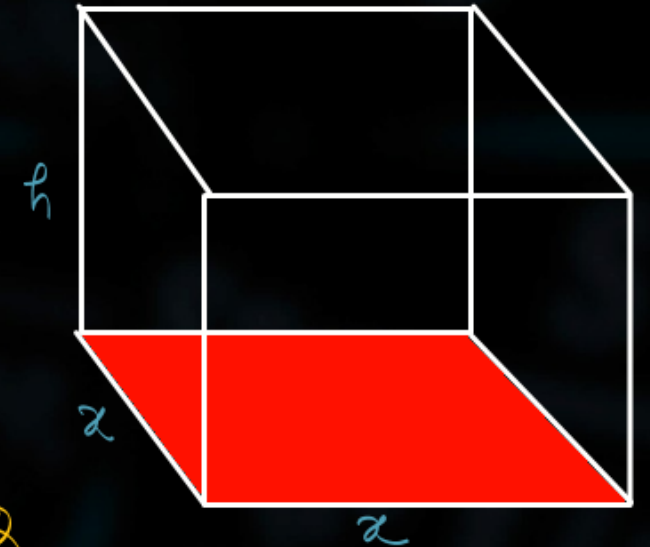
$$f''(x) = -4V \left(\frac{-2}{x^3} \right) + 2$$

$$= \frac{8V}{x^3} + 2$$

$$= \frac{8x^2 h}{x^3} + 2$$

$$= \frac{48h}{2x} + 2$$

$$= 6 > 0 \quad f(x) \text{ min}$$



$$BA \times 70 + C.S.A \times 45$$

$$xy + 20 + 2h(x+y) \times 45 = 4 \times 70 + 2 \times 2(2+2) \times 45 = 280 + 720 = 1000 \text{ RS}$$

45
+16

A tank with rectangular base and rectangular sides, open at the top is to be constructed, so that its depth is 2 m and volume is 8 m³. If building of tank costs ₹70 per sq m for the base and ₹45 per sq m for sides. What is the cost of least expensive tank?

$$1 \text{ m}^2 = 70$$

$$V = x \times y \times h = 84 = x \times y \times 2 \Rightarrow xy = 4 \quad y = \frac{4}{x}$$

$$\begin{aligned} S.A &= 2h(x+y) + xy \\ &= 2 \times 2(x+y) + xy \\ &= 4 \times \left[x + \frac{4}{x} \right] + 4 \end{aligned}$$

$$f(x) = 4 \left(x + \frac{4}{x} \right) + 4$$

$$f'(x) = 4 \left[1 + 4 \times \frac{-1}{x^2} \right] + 0$$

$$4 \left(1 - \frac{4}{x^2} \right) = 0$$

$$1 - \frac{4}{x^2} = 0$$

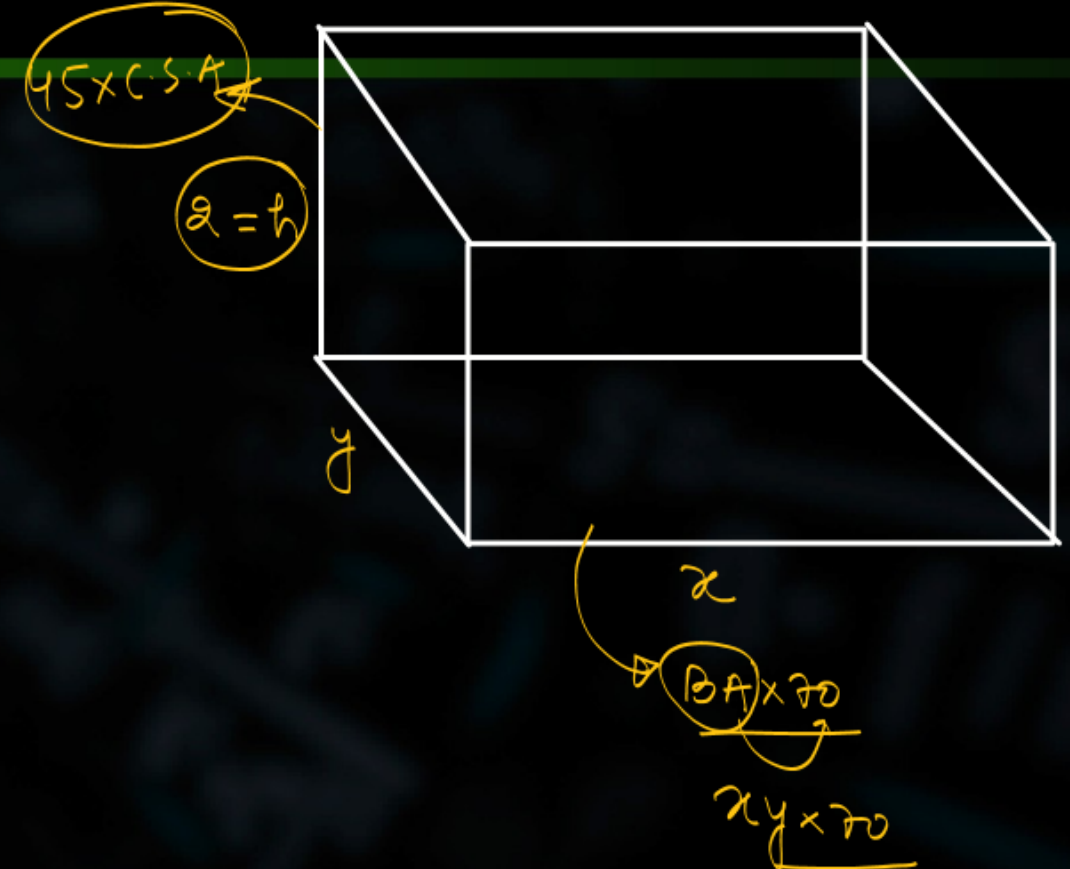
$$1 = \frac{4}{x^2}$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x = 2$$

$$y = 2$$



Prove that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

$$V_{\text{cyl.}} = \pi r^2 h$$

$$V = \pi \left[R^2 - \frac{h^2}{4} \right] h$$

$$= \pi \left[R^2 h - \frac{h^3}{4} \right]$$

$$f(h) = \pi \left[R^2 h - \frac{h^3}{4} \right]$$

$$f'(h) = \pi \left[R^2 - \frac{3h^2}{4} \right]$$

$$R^2 - \frac{3h^2}{4} = 0 \Rightarrow R^2 = \frac{3h^2}{4}$$

$$\frac{4R^2}{3} = h^2$$

$$\sqrt{\frac{4R^2}{3}} = h$$

$$\frac{2R}{\sqrt{3}} = h$$

$$f''(h) = \pi \left[0 - \frac{3}{4} \times 2h \right]$$

$$= -\pi \times \frac{3}{4} \times 2h$$

$$= -\frac{3\pi h}{2} < 0$$

$f(h)$ is max.

$$V_{\text{cyl.}} = \pi r^2 h$$

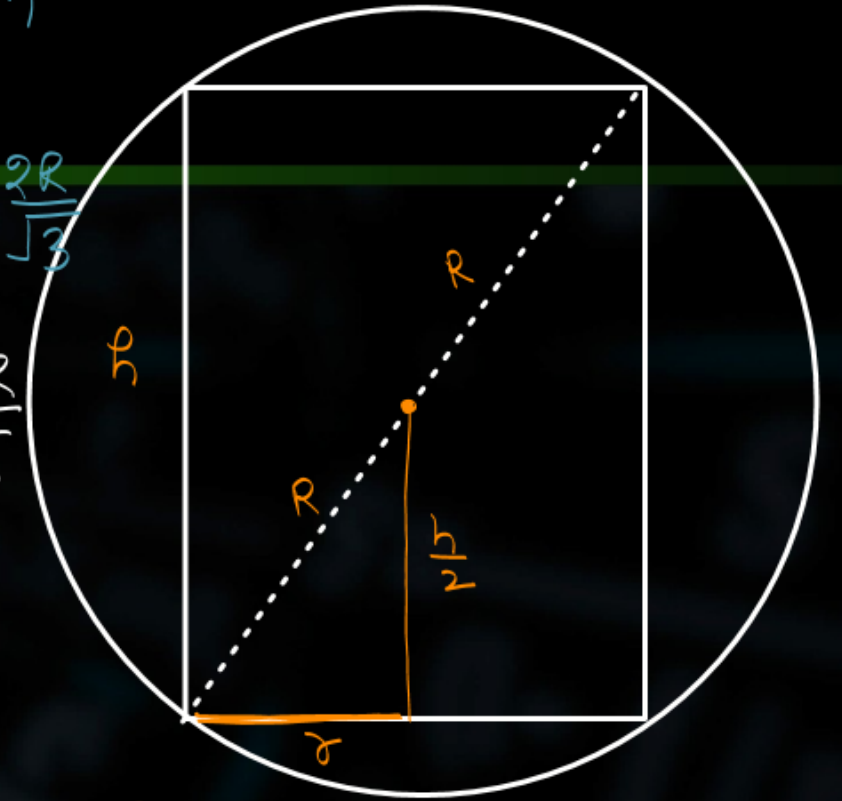
$$= \pi \left[R^2 - \frac{h^2}{4} \right] h$$

$$= \pi \left[R^2 - \frac{4R^2}{3 \times 4} \right] \times \frac{2R}{\sqrt{3}}$$

$$\Rightarrow \pi \left[\frac{3R^2 - R^2}{3} \right] \frac{2R}{\sqrt{3}}$$

$$\Rightarrow \pi \times \frac{2R^2}{3} \times \frac{2R}{\sqrt{3}}$$

$$\Rightarrow \frac{4\pi R^3}{3\sqrt{3}}$$



$$R^2 = r^2 + \left(\frac{h}{2}\right)^2$$

$$R^2 = r^2 + \frac{h^2}{4}$$

$$R^2 - \frac{h^2}{4} = r^2$$

Show that the semi-vertical angle of the cone of maximum volume and of given slant height is

$$\cos^{-1} \frac{1}{\sqrt{3}}$$

$$l^2 = h^2 + r^2 \Rightarrow \underline{l^2 - h^2 = r^2}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi [l^2 - h^2] h$$

$$= \frac{1}{3} \pi [l^2 h - h^3]$$

$$f(h) = \frac{\pi}{3} [l^2 h - h^3]$$

$$f'(h) = \frac{\pi}{3} [l^2 - 3h^2]$$

$$\frac{\pi}{3} [l^2 - 3h^2] = 0$$

$$l^2 - 3h^2 = 0$$

$$l^2 = 3h^2$$

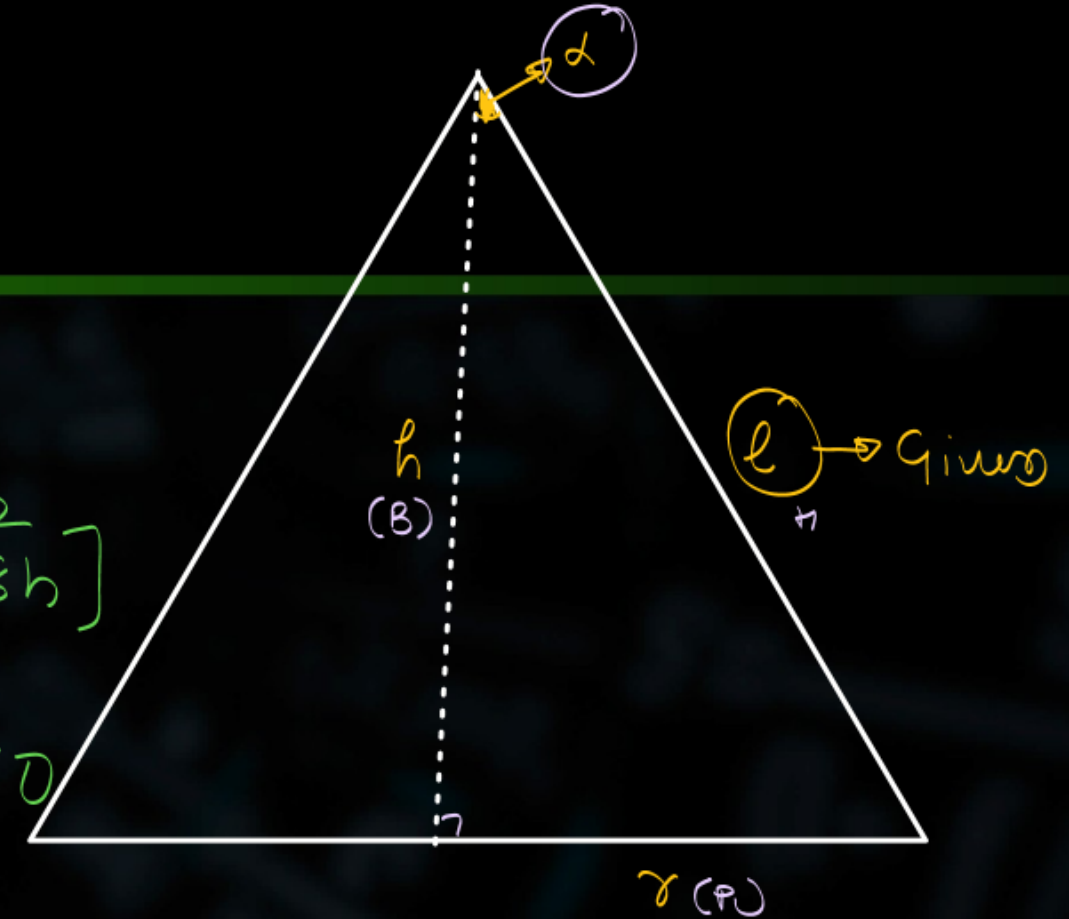
$$l = \sqrt{3}h$$

$$\alpha = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$f''(h) = \frac{\pi}{3} [0 - 6h]$$

$$= -2\pi h < 0$$

$f(h)$ is Max.



$$\cos \alpha = \frac{h}{l}$$

$$\cos \alpha = \frac{h}{h\sqrt{3}}$$

$$\alpha = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\rightarrow A.R = \text{Max.} = 2\alpha \times 2\beta = 2 \times \frac{a}{\sqrt{2}} \times 2 \times \frac{b}{\sqrt{2}} = \frac{2ab}{2} = ab$$

Find the area of the greatest rectangle that can be inscribed in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Length of Rect.} = 2\alpha \quad \text{Width of Rect.} = 2\beta$$

$$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$$

$$\frac{\alpha^2}{a^2} = 1 - \frac{\beta^2}{b^2}$$

$$\frac{\alpha^2}{a^2} = \frac{b^2 - \beta^2}{b^2}$$

$$\alpha^2 = \frac{a^2}{b^2} (b^2 - \beta^2)$$

$$\alpha = \frac{a}{\sqrt{2}}$$

$$\text{Area of Rect.} = 2\alpha \times 2\beta = 4\alpha\beta$$

$$f(\beta) = 4\beta \sqrt{\frac{a^2}{b^2} (b^2 - \beta^2)}$$

$$= 4\beta \times \frac{a}{b} (b^2 - \beta^2)^{\frac{1}{2}}$$

$$f(\beta) = \frac{4a}{b} \beta (b^2 - \beta^2)^{\frac{1}{2}}$$

$$f'(\beta) = \frac{4a}{b} \left[1 \times (b^2 - \beta^2)^{\frac{1}{2}} + \frac{1}{2} (b^2 - \beta^2)^{-\frac{1}{2}} \times (0 - 2\beta) \times \beta \right]$$

$$= \frac{4a}{b} \left[(b^2 - \beta^2)^{\frac{1}{2}} - \frac{\beta^2}{(b^2 - \beta^2)^{\frac{1}{2}}} \right] = 0$$

$$(b^2 - \beta^2)^{\frac{1}{2}} = \frac{\beta^2}{(b^2 - \beta^2)^{\frac{1}{2}}}$$

$$\sqrt{b^2 - \beta^2} \times \sqrt{b^2 - \beta^2} = \beta^2$$

$$b^2 - \beta^2 = \beta^2$$

$$b^2 = 2\beta^2$$

$$\frac{b^2}{2} = \beta^2 \Rightarrow \beta = \frac{b}{\sqrt{2}}$$



$$\begin{aligned} & \rightarrow 2ab \\ A &= 2\alpha \times 2\beta \\ &= 4\alpha\beta \end{aligned}$$

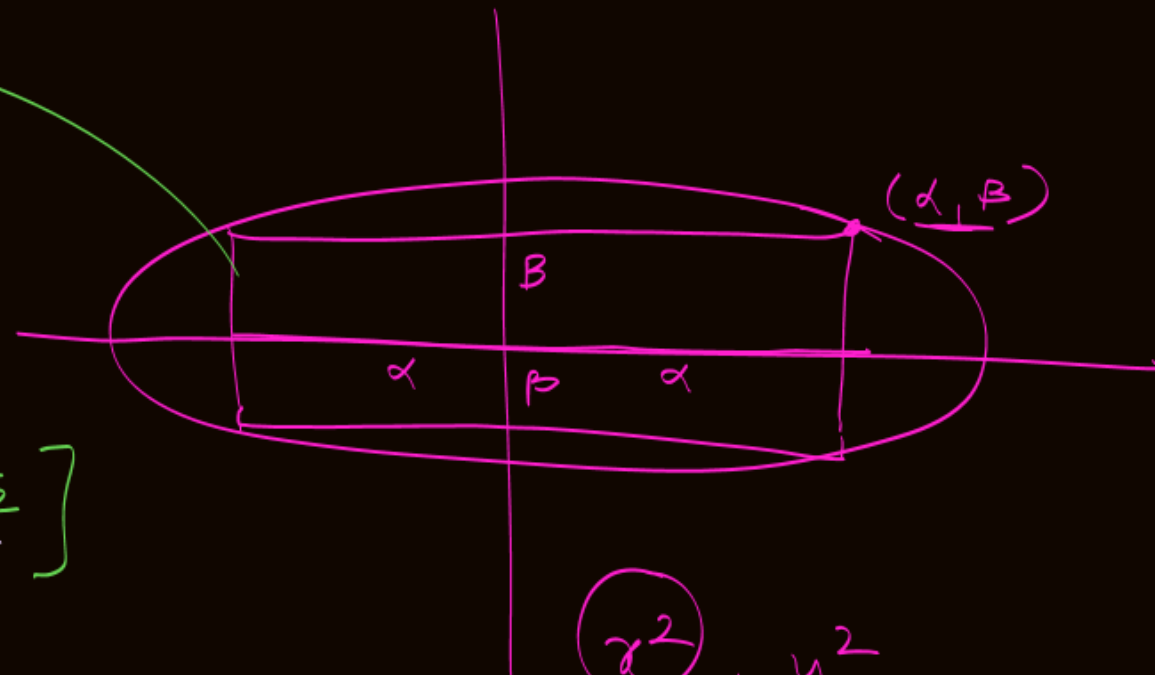
$$= 4\beta \times \frac{a}{b} \sqrt{b^2 - \beta^2}$$

$$= \frac{4a}{b} \left[\beta \times (b^2 - \beta^2)^{\frac{1}{2}} \right]$$

$$= \frac{4a}{b} \left[\sqrt{b^2 - \beta^2} - \frac{\beta^2}{\sqrt{b^2 - \beta^2}} \right] = 0$$

$$\beta = \frac{b}{\sqrt{2}}$$

$$\alpha = \frac{a}{\sqrt{2}}$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$$

$$\alpha = \frac{a}{\sqrt{2}} \sqrt{b^2 - \beta^2}$$

Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

$$A = \text{Max}$$

$$A = \frac{1}{2} \times P \times B$$

$$= \frac{1}{2} \times P \times (H^2 - P^2)^{\frac{1}{2}}$$

$$f(P) = \frac{1}{2} \left[P \times (H^2 - P^2)^{\frac{1}{2}} \right]$$

$$f'(P) = 0$$

$$P = B$$

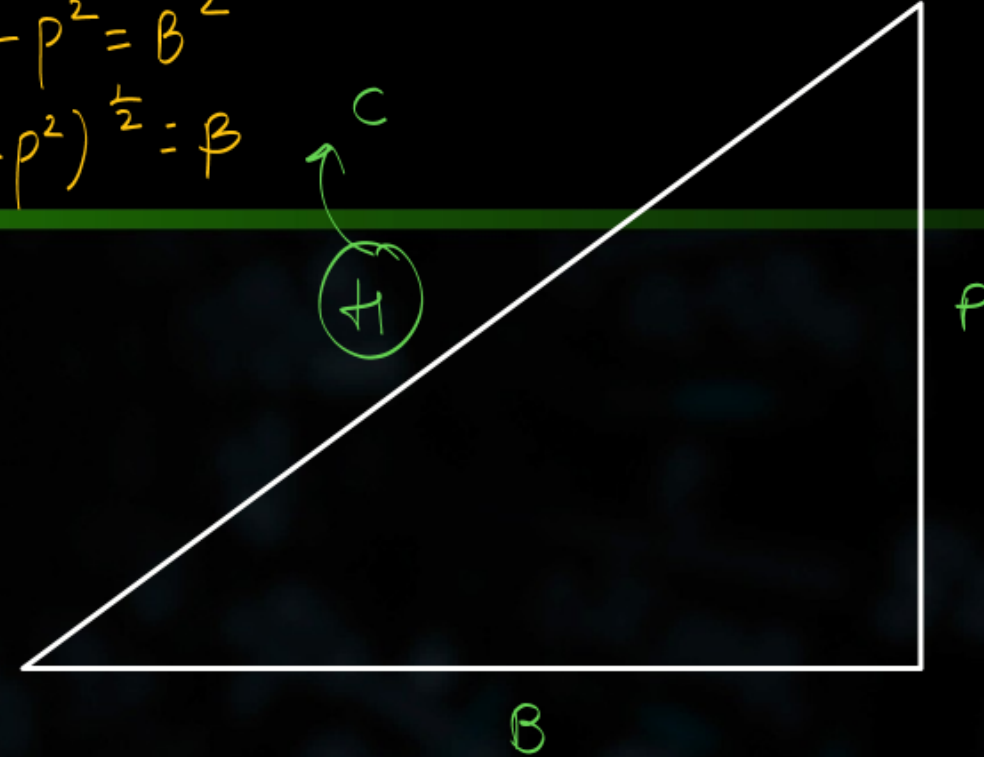
$$\Delta ABC \rightarrow \text{I.S.S.}$$

$$P = B$$

$$H^2 = B^2 + P^2$$

$$H^2 - P^2 = B^2$$

$$(H^2 - P^2)^{\frac{1}{2}} = B$$



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